Master's thesis

NTNU
Norwegian University of Science and Technology
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The Magic Formula and the Fama and French Five Factor Model

An Empirical Study on Value Investing and Market Anomalies in the US Market

Master's thesis in Financial Economics
Supervisor: Svein-Arne Persson
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Abstract

This master's thesis conducts a comprehensive evaluation of Joel Greenblatt's Magic Formula, as described in his influential book 'The Little Book That Beats the Market', applying it retrospectively as an investment strategy in the U.S. stock market from 1998 to 2023 across top market capitalizations of 3500, 2500, 1500, and 500 companies. The study analyzes returns using the Fama and French Five Factor Model, which includes market, size, value, profitability, and investment risk factors. It reveals a distinct trend: as the investment universe narrows, returns decrease, underscoring the importance of diversification across various company sizes, not just the largest firms. The research identifies significant shifts in the impact of specific risk factors. Notably, the size risk factor is crucial within the largest group of 3500 companies, while the value and aggressive investment styles significantly influence returns in the top 500 companies' segment. Profitability emerges as a consistent and key determinant of returns across all segments. These factors represent risk premiums that enhance investor returns above the market average. Impressively, the top 3500 companies achieved a robust annualized return of 20.7%, outperforming the market benchmark, with the top 500 companies recording an 11.3% return. This thesis enhances our understanding of how the Magic Formula can outperform the market, providing higher risk-adjusted returns across various market capitalizations.

Keywords:

Value Investing, Joel Greenblatt, Magic Formula, Fama-French Five Factor Model, Market Capitalization Segments, US Stock Market, Portfolio Analysis, Market Risk Factors, Diversification Benefits, Investment Performance, Risk-Adjusted Returns, Quantitative Investment Strategy, Market Efficiency, Market Anomalies, Factor Investing, Asset Pricing Model

Sammendrag

Denne masteroppgaven utfører en grundig evaluering av Joel Greenblatts Magic Formula, en investeringsstrategi beskrevet i hans bok 'The Little Book That Beats the Market', med backtesting i det amerikanske aksjemarkedet fra 1998 til 2023 over ulike markedsverdisegmenter: de største 3500, 2500, 1500, og 500 selskapene. Studien analyserer avkastning ved bruk av Fama og Frenchs femfaktormodell, som inkluderer markeds-, størrelse-, verdi-, lønnsomhets- og investeringsrisikofaktorer. Den avdekker en tydelig trend: etter hvert som investeringsuniverset innsnevres, reduseres avkastningen, noe som understreker viktigheten av diversifisering på tvers av forskjellige selskapsstørrelser, ikke bare investere i de største børsnoterte selskapene. Forskningen identifiserer betydelige endringer i påvirkningen av spesifikke risikofaktorer. Størrelsesfaktoren er avgjørende i den største gruppen på 3500 selskaper, mens verdifaktoren og en aggressiv investeringsstil har betydelig innflytelse på avkastningen i segmentet av de 500 største selskapene. Lønnsomhet fremstår som en avgjørende determinant for avkastning på tvers av alle utforskede segmenter. Disse faktorene representerer risikopremier som gir investorenes avkastning utover markedsnivået. Imponerende nok oppnår de 3500 største selskapene en årlig avkastning på 20,7 %, som overgår markedsreferansen, mens de 500 største selskapene oppnår en avkastning på 11,3 %. Denne oppgaven utvider vår forståelse av hvordan Magic Formula kan overgå markedet, og tilby høyere risikojusterte avkastninger på tvers av ulike markedsverdier.

Nøkkelord:

Verdiinvestering, Joel Greenblatt, Magic Formela, Fama-French Femfaktormodell, Porteføljeanalyse, Markedsrisikofaktorer, Diversifisering, Risikojusterte Avkastninger, Markedseffektivitet, Markedsanomalier, Faktorinvestering

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1 Introduction

The world of financial markets is ever-changing, shaped by new strategies and ideas that challenge the status quo. This thesis investigates the performance of Joel Greenblatt's Magic Formula in the context of the Fama and French Five Factor Model, focusing on the U.S. stock market from 1998 to 2023. It aims to understand if the Magic Formula's ability to pick undervalued stocks is due to taking on more risk as defined by the five-factor model, which includes measures of market, size, value, profitability, and investment risks.

The study evaluates the effectiveness of the Magic Formula at identifying stocks priced below their intrinsic value and examines whether the excess returns attributed to this strategy are compensated by higher risk exposure. By doing so, it offers insights into the underlying mechanisms through which the Magic Formula operates and validates the relevance of value investing in modern volatile market conditions. This research is vital for both academic scholars and practical investors seeking to comprehend the factors that lead to better market returns.

1.1 Background and Motivation

Value investing, originating from the foundational works of Benjamin Graham and David Dodd (1934), has long been heralded as a strategy capable of withstanding the test of time, demonstrating strong potential for generating robust investment returns. Unlike growth investing, which targets companies expected to experience rapid growth in revenues or earnings, value investing targets firms that are trading below their intrinsic value. This intrinsic value is often assessed using metrics related to profitability and the return on employed capital. Joel Greenblatt's Magic Formula provides a systematic approach to stock selection by focusing on earnings yield and return on capital, aiming to identify undervalued companies with strong operational efficiency (Greenblatt, 2010). The Fama and French Five Factor Model complements this approach by integrating size, value, profitability, and investment factors with market risk to explain equity returns, offering a comprehensive framework for asset pricing (Fama & French, 2015).

The analysis presented by Greenblatt for the period between 1988 and 2009, which centers on the top decile of the largest 2500 companies in the U.S., documented an annualized return of 15.2%. Yearly return for the market-weighted index S&P 500 during the same period was 9.5% geometric average. Extending the investigation to the period from 1998 to 2023, the study explores the performance of the Magic Formula within the same scope. It is found that the strategy achieved an annualized return of 17.0% for top 2500 companies, markedly outstripping the equally weighted market return of 9.9% for the largest 2500 companies during the corresponding timeframe, generating an excess return of 7.1%. The geometric average yearly return for S&P500 last 25 years is 8.3%.

Aligning the scope with Greenblatt's initial study ensures a direct comparison, bolstering the validity of the comparative analysis. The findings contribute to the discourse on value investing by examining the performance of the Magic Formula over an extended period, thereby offering insights into the persistence of the strategy's effectiveness and the evolving influence of market factors.

1.2 Research Questions

This study is guided by two principal research questions:

1) How does the Magic Formula perform in the top 3500 and top 500 market capitalization segments of the US stock market from 1998 to 2023, and what factors contribute to its performance in these segments?

2) To what extent do the Fama and French Five Factors explain the returns of the Magic Formula, and what implications does this have for understanding market efficiency and risk premiums?

1.3 Objectives and Scope

The principal objective of this research is to conduct a comprehensive analysis of the performance of Greenblatt's Magic Formula, assessing its effectiveness when analyzed through the Fama and French Five Factor Model over a 25-year period from 1998 to 2023. This study aims to determine to what extent the model's factors, including market premium (EMR), size premium (SMB), value premium (HML), profitability premium (RMW), and investment premium (CMA), can account for the returns achieved by the strategy. A detailed exploration of each factor will be provided in subsequent chapters, enhancing the understanding of their distinct impacts on stock returns.

While Greenblatt's original analysis focused on the largest 2500 US companies, this study expands the scope to include a wider spectrum of companies by examining different subsets, specifically the top 3500, 2500, 1500, and 500 companies by market capitalization. This broader scope enables a more nuanced examination of how company size and market factors contribute to investment returns within the US market.

1.4 Methodology

This thesis employs a quantitative approach through regression analysis, aiming to explore the nuanced relationship between Joel Greenblatt's Magic Formula and the Fama and French Five Factor Model across the US stock market. Utilizing the *statsmodels* library in Python, this study constructs a regression model to analyze the performance of the Magic Formula, particularly focusing on how the identified factors—EMR (Excess Market Return), SMB (Small Minus Big), HML (High Minus Low), RMW (Robust Minus Weak), and CMA (Conservative Minus Aggressive)—affect the strategy's returns from 1998 to 2023.

The model assesses returns across diverse portfolio segments, delineated by company size, to determine the explanatory power of the Five Factor Model on the performance of value investing strategies. This approach allows for a comprehensive examination of each factor's impact, guided by the significance of their coefficients and the variance in R-squared values across different market segments.

This methodology section lays the groundwork for a comprehensive analysis, introducing the foundational elements essential for the study. A deeper dive into the specifics, including data collection, processing, and factor generation, as well as the detailed statistical methods used for analysis, will be further elaborated in Chapter 3. This structured approach ensures a thorough understanding of the dynamics at play in evaluating the effectiveness of the Magic Formula through the Fama and French Five Factor Model.

1.5 Structure of the Thesis

The structure of the thesis is designed to guide the reader through the research process systematically. After the introduction, Chapter 2 provides a thorough literature review, setting the stage for the study by discussing the theoretical underpinnings and existing research in the field. Chapter 3 describes the data collection and methodology, detailing the steps taken to ensure a rigorous analysis. Chapter 4 presents the empirical findings, while Chapter 5 contextualizes these results within the broader body of literature and explores their implications. The final chapter, Chapter 6, concludes the thesis by summarizing the significant contributions of the research, recognizing its limitations, and proposing avenues for future investigation. Supporting information is included in the appendices, and references are cited to underpin the research.

2 Literature Review

This chapter delves into the core theories and principles that form the foundation of this thesis. It critically examines the Efficient Market Hypothesis (EMH), Adaptive Market Hypothesis (AMH), behavioral finance, delves into the tenets of value investing and fundamental analysis, explores Joel Greenblatt's Magic Formula, traces the evolution of factor models in finance, and discusses the comprehensive Fama and French Five Factor Model. By scrutinizing these theories and their developments, this review not only constructs the theoretical framework vital for the subsequent empirical analysis but also contextualizes the study within the broader discourse on market efficiency, value investing, and asset pricing models.

2.1 Efficient Market Hypothesis

The Efficient Market Hypothesis (EMH), articulated by Eugene Fama in the 1960s, posits that stock prices fully reflect all available information, making it theoretically impossible for investors to consistently achieve above-average returns through information asymmetry (Fama, 1970). EMH categorizes market efficiency into three levels — weak, semi-strong, and strong — each reflecting the degree of information reflected in stock prices. Despite its pivotal role in finance, EMH has been rigorously challenged, especially in the context of behavioral finance, which highlights the impact of psychological factors on investor decisions and market dynamics.

Burton Malkiel, in his influential works, notably "A Random Walk Down Wall Street" (Malkiel, 2003), provides a nuanced perspective on EMH. While Malkiel supports the general premise that markets are highly efficient in reflecting information, he also acknowledges the existence of market anomalies that cannot be fully explained by EMH. His balanced view suggests that while EMH holds under many conditions, there are instances where behavioral biases and irrational investor behaviors create market inefficiencies that can be exploited.

Furthermore, the critique by Robert A. Haugen adds depth to this discussion, as he presents evidence of predictable inefficiencies in the market, directly challenging the assumptions of EMH (Haugen, 1995). Haugen's findings suggest that despite the general efficiency of markets, there are systematic patterns and anomalies that discerning investors can leverage for potential gains.

2.2 Adaptive Market Hypothesis

Building on the insights from EMH and its critiques, the Adaptive Market Hypothesis (AMH) proposed by Andrew Lo in 2004 suggests that market efficiency is not a static condition, but a dynamic process influenced by the behaviors of market participants (Lo, 2004). AMH incorporates principles from evolutionary biology, positing that as conditions and behaviors evolve, so does the degree of market efficiency. This hypothesis provides a flexible framework for understanding how financial markets can exhibit varying degrees of efficiency over time, shaped by the adaptive responses of investors to new information and market conditions.

2.3 Behavioral Finance

Behavioral finance examines how psychological influences, including irrational behavior and cognitive biases, impact financial decisions and market outcomes. This field offers critical insights into deviations from the traditional model of the rational investor, as outlined by EMH. Pioneering research by Daniel Kahneman and Amos Tversky introduced the concept of cognitive biases such as overconfidence and loss aversion, which systematically affect investor behavior and can lead to predictable but irrational financial behaviors (Kahneman & Tversky, 1979). This field provides critical insights into why markets might not be as efficient as posited by EMH and AMH, offering context for observed market anomalies.

2.4 Value Investing and Fundamental Analysis

After establishing the limitations of EMH through behavioral insights, this section explores value investing and fundamental analysis. These investment strategies seek to capitalize on market inefficiencies by identifying undervalued stocks that the market has not priced correctly. Rooted in the principles set forth by Benjamin Graham and David Dodd, value investing involves detailed analysis of a company’s financial statements to determine its true intrinsic value, contrasting significantly with the EMH assertion that all securities are fairly priced at all times (Graham & Dodd, 1934). This investment philosophy leverages fundamental analysis, a rigorous evaluation of a company’s financials, management efficacy, competitive advantages, and market conditions to ascertain its true economic worth.

In practice, fundamental analysis involves a comprehensive assessment of financial statements, earnings stability, asset management efficiency, and future growth prospects, among other metrics. The aim is to develop a detailed understanding of a company's real value independent of its current market price. Value investing is predicated on the belief that markets are not fully efficient and often misprice securities relative to their underlying economic value due to overreactions to transient news events.

Value investing is better understood through an example. If the market price of a company's stock is $100, but a detailed fundamental analysis suggests that the stock's intrinsic value is $150 based on its financial health and growth prospects, this discrepancy presents a buying opportunity. Such situations often arise during periods of market volatility, where pervasive pessimism may undervalue fundamentally strong companies. A value investor, recognizing the stock’s undervaluation, would purchase shares at the depressed price, anticipating a correction in the market’s valuation as other investors gradually recognize the company's true value.

Despite the prevailing efficient market hypothesis (EMH), which posits that stock prices fully reflect all available information, the practice of value investing provides a compelling argument against complete market efficiency. It posits that through diligent analysis and adopting a long-term investment perspective, it is possible to exploit market inefficiencies for substantial returns. This approach not only challenges the foundations of EMH but also underscores the potential of value investing to capitalize on discrepancies between a company’s market price and its fundamental value.

2.5 Joel Greenblatt’s Magic Formula

Joel Greenblatt's Magic Formula, articulated in his widely acclaimed work, "The Little Book That Beats the Market" (2006), which became a New York Times bestseller, and its follow-up, "The Little Book That Still Beats the Market" (2010), outlines a systematic investment strategy designed to identify high-quality firms at undervalued prices. This strategy embodies a sophisticated approach to value investing, leveraging precise financial metrics to uncover investment opportunities that typically remain unnoticed by conventional market analyses.

The Magic Formula simplifies the investment process while maintaining a robust analytical framework, making it accessible to both novice investors and seasoned professionals. By focusing on acquiring undervalued companies that demonstrate operational efficiency, Greenblatt's strategy challenges traditional investment perspectives and offers a practical methodology for achieving superior long-term returns.

The conceptual underpinnings of the Magic Formula rest on the hypothesis that markets, while generally efficient, are prone to episodic inefficiencies that result in the mispricing of securities. Greenblatt contends that a high Return on Capital (ROC) is indicative of firms that efficaciously leverage their capital to generate profits, denoting operational excellence. Conversely, a high Earnings Yield (EY) suggests that a company's stocks are undervalued relative to its earnings, representing a lucrative investment opportunity. The synthesis of these metrics aims to systematically exploit market inefficiencies by identifying firms that are both financially robust and underpriced.

Two factors define the key components of the Magic Formula:

1. **Return on Capital (ROC)**:
   Return on Capital measures a firm's efficiency in generating pre-tax earnings from its capital employed. This metric serves as a measure of a firm's ability to convert its capital into profits, indicating operational excellence. Return on Capital (ROC) is defined as

   $$\frac{EBIT}{Net\ Working\ Capital + Net\ Fixed\ Assets}$$

   The calculation generates a factor by looking at the ratio of Earnings Before Interest and Tax (EBIT) over employed capital, which includes both working capital and fixed assets.

2. **Earnings Yield (EY)**:
   Calculated as EBIT over Enterprise Value, where Enterprise Value includes the market value of equity and net interest-bearing debt. Earnings Yield assesses how much earnings a company produces per unit of valuation, acting as the reciprocal of a valuation multiple. A high EY indicates that a company's stocks are undervalued relative to its earnings, suggesting a profitable investment opportunity. Earnings Yield (EY) is defined as

   $$\frac{EBIT}{Enterprise\ Value}$$

   The second factor is a ratio of Earnings Before Interest and Tax (EBIT) over Enterprise Value (EV).

By combining Return on Capital and Earnings Yield, the Magic Formula creates a composite ranking to facilitate portfolio selection. Companies are ranked into deciles, with the top 10% of companies, based on these combined rankings, expected to perform well, and the bottom 10% anticipated to underperform. This decile-based ranking system is updated quarterly to reflect the latest financial data and market valuations, enabling a dynamic approach to investment that seeks to capitalize on market discrepancies between price and intrinsic value. The Magic Formula utilizes two critical financial metrics, Return on Capital (ROC) and Earnings Yield (EY), reflecting a company's operational efficiency and valuation, for creating the composite ranking.

The composite ranking process involves the following steps:

1. **Calculation and Ranking**:
   Each company within the investment universe is calculated for ROC and EY and then ranked based on each metric separately. The rankings reflect the company's standing relative to its peers, with lower rankings indicating better performance.

2. **Score Addition**:
   The individual ranks for ROC and EY are added for each company to form a composite score. This score is crucial as it combines insights from both operational efficiency and valuation metrics, providing a balanced view of a company's investment potential.

3. **Decile Sorting**:
   Companies are sorted based on their composite scores into deciles. The first decile contains the top 10% of companies with the lowest composite scores, identified as having the highest potential for performance. In contrast, the tenth decile includes the bottom 10% with the highest composite scores, flagged as likely underperformers.

4. **Dynamic Updates**:
   This decile-based ranking system is updated quarterly, incorporating the latest financial data and adjustments in market valuations. This ensures that the investment recommendations remain relevant and timely, adjusting for any changes in economic conditions or company performance.

To facilitate understanding of the composite ranking procedure, consider the following example. If Company X has a ROC rank of 20 and an EY rank of 15 out of 1000, its composite score would be 35. The companies are then ordered based on their composite scores, from best to poorest. The top 10% of these scores, representing the most favorable investment prospects, fall into the 1st decile. They are the best due to their optimal balance of profitability and valuation.

The Magic Formula's strategy is predicated on the hypothesis that, while markets are generally efficient, they are occasionally prone to inefficiencies that lead to the mispricing of securities. The methodical integration of ROC and EY aims to systematically exploit these inefficiencies by identifying firms that are both operationally efficient and financially undervalued. This approach challenges the efficient market hypothesis and highlights the formula's potential to yield superior returns through disciplined stock selection and quantitative analysis.

The robustness of Joel Greenblatt's Magic Formula is further substantiated by numerous studies that confirm its effectiveness across different markets and time periods. For instance, research comparing various value investment strategies in the Finnish market from 1991 to 2013 highlights that the Magic Formula strategy not only consistently outperformed the market but also provided substantial risk-adjusted returns (Davydov et al., 2016). Additionally, Håkansson and Kvarnmark (2016) investigated the application of the Magic Formula on the Nordic stock market from 2007 to 2016, finding that the compound annual growth rate (CAGR) of the magic formula portfolio was 16.6% compared to 1.4% for the OMX Nordic 40. Their study also highlighted that the Magic Formula portfolio had lower volatility and a higher Sharpe ratio, indicating better risk-adjusted performance. Moreover, their results were not solely attributable to taking on excess risk, as confirmed by applying the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model.

Furthermore, Gustavsson and Strömberg (2017) studied the Magic Formula on the Swedish stock market from 2007 to 2017 and found that it achieved an average annual return of 21.25%, significantly outperforming the OMXS30, which had a return of 5.22%. They also found that the Magic Formula had a higher Sharpe ratio, suggesting better risk-adjusted returns. A comprehensive study of the Magic Formula in the Benelux market from 1995 to 2014 found that the formula outperformed the market by an average of 7.7% annually, without taking on additional risk, indicating its efficacy beyond the U.S. market (Kukkasniemi, 2013).

Hardeman (2020) conducted an analysis of the Magic Formula on the Euronext Brussels, which showed that the strategy could achieve abnormal returns even in a smaller, less liquid market. Kreft (2022) extended the analysis to the period from 1987 to 2021, finding that the Magic Formula was effective at generating alpha until 2010, but its effectiveness diminished in the following decade. Additionally, the empirical study by Vestre and Wikheim (2022) tested the Magic Formula on the Oslo Stock Exchange from 2003 to 2022. They found that the Magic Formula generated a compound annual growth rate (CAGR) of

21.56%, compared to the benchmark's 13.54%. Färdig and Hammarling (2016) studied the Magic Formula on the Stockholm Stock Exchange from 2005 to 2015 and found that it achieved an average annual return of 19.37%, outperforming the market index at 9.44%, demonstrating better risk-adjusted returns. Persson and Selander (2009) backtested the Magic Formula in the Nordic region from 1998 to 2008 and found a CAGR of 14.68%, compared to 9.28% of the MSCI Nordic, further supporting the formula's effectiveness. And Gustafsson and Selling (2014) found that Greenblatt’s Magic Formula generated an excess yield of 15% above the market index on the Swedish stock market, with a low standard deviation.

These findings collectively affirm that the Magic Formula is not only practical but also theoretically sound, capable of discerning undervalued, high-quality stocks poised for appreciable gains. This dual emphasis on operational efficiency and attractive pricing enhances the formula’s utility in navigating complex market dynamics, making it a critical area of study for scholars and investors interested in the intricacies of value investing and market efficiency.

2.6 The Birth of Factor Models in the 1990s

The Fama and French model (Fama & French, 1993), initially introduced with three factors in 1993, marked a significant evolution in understanding the complexities of stock returns beyond what the Capital Asset Pricing Model (CAPM) could explain (Sharpe, 1964). This model extends the CAPM by incorporating two additional factors—size and value—alongside the market risk factor to better account for differences in stock returns.

This development was followed by Professor Robert Haugen's introduction in 1996 of an extensive list of 71 factors that potentially influence stock returns, expanding the scope of factor-based analysis even further (Haugen & Baker, 1996). These factors were found to outperform the market, yielding a return of 12.55% compared to the market's 9.38% over the period from 1994 to 2004 (Greenblatt, 2006).

Interestingly, Joel Greenblatt's Magic Formula investing strategy, which uses only two factors—earnings yield and return on capital—did even better, generating a return of 18.43% over the same period (Greenblatt, 2006). This suggests that while a comprehensive factor model can improve performance, a carefully selected, simpler model can also be highly effective.

The Fama and French model, introduced in 1993, posits that three factors influence stock returns (Fama & French, 1993):

1. **Excess Market Return (EMR)**: The return of the market portfolio over the risk-free rate, representing the general market risk.
2. **Size Premium (SMB - Small Minus Big)**: The additional return investors can expect from investing in companies with smaller market capitalizations compared to those with larger capitalizations.
3. **Value Premium (HML - High Minus Low)**: The excess returns of stocks with high book-to-market ratios (value stocks) over those with low book-to-market ratios (growth stocks).

Shortly after, Mark Carhart introduced the momentum factor in 1997, which has since been recognized as the fourth critical factor in asset pricing models (Carhart, 1997):

- **Momentum (MOM)**: This factor accounts for the tendency of stocks that have performed well in the past to continue performing well in the near future, and vice versa for stocks that have performed poorly.

In 2015, Fama and French expanded their model to include two more factors (Fama & French, 2015):

4. **Profitability (RMW - Robust Minus Weak)**: This factor reflects the difference in returns between companies with high profitability and those with low profitability.
5. **Investment (CMA - Conservative Minus Aggressive)**: This factor measures the return difference between firms that are conservative in their investment activities versus those that are aggressive.

These advancements in the 1990s and beyond significantly broadened the understanding of stock returns and laid the groundwork for the sophisticated factor models used in finance today.

2.7 The Evolution of Factor Models

The finance landscape has profoundly benefited from the advent of factor models, designed to elucidate the variances in stock returns that are not captured by market indices alone. The Capital Asset Pricing Model (CAPM), introduced by William F. Sharpe (1964), stands as a foundational model in this realm. CAPM proposes that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium, which is based on the beta (β) of that security or portfolio. The equation is given by:

$$R_{i,t} = R_{f,t} + \beta_i \left(R_{m,t} - R_{f,t}\right) + \epsilon_{i,t}$$

Where:

- $R_{i,t}$ is the return on asset i at time t,
- $R_{f,t}$ is the risk-free rate of return at time t,
- β_i represents the beta of the asset i, indicating its sensitivity to market movements,
- $R_{m,t}$ is the return of the market at time t,
- $\epsilon_{i,t}$ is the error term for asset i at time t, capturing the asset-specific, unsystematic risk not explained by the market's movements.

Despite its widespread adoption, CAPM's limitation in fully explaining stock returns prompted the development of more comprehensive models. A significant milestone in this evolution was the introduction of the Fama and French Three Factor Model (Fama & French, 1993). Expanding upon CAPM, this model integrates two additional factors—size and value—into the asset pricing equation to account for differences in stock returns that

CAPM's market risk factor alone could not explain. The French Three Factor Model is formalized as follows:

$$R_{i,t} - R_{f,t} = \alpha + \beta_m (MKT_t - R_{f,t}) + \beta_s SMB_t + \beta_v HML_t + \epsilon_{i,t}$$

Where:

- $R_{i,t} - R_{f,t}$ represents the excess return of the asset i over the risk-free rate R_f at time t,
- α is the intercept, capturing the average excess return unexplained by the model,
- β_m, β_s, and β_v represent the sensitivity of the asset i's returns to the market excess returns, size premium, and value premium, respectively,
- SMB_t and HML_t are the size and value factors at time t,
- $\epsilon_{i,t}$ is the error term for asset i at time t, capturing the return variance not explained by the three factors.

2.8 Carhart Four-Factor Model

Amidst the evolving landscape of factor models, the Carhart Four-Factor Model emerges as a pivotal extension of the Fama and French framework, introducing momentum as a significant determinant of stock returns. Developed by Mark Carhart (1997), this model adds a fourth factor, momentum (MOM), to the original three factors of market excess return, size, and book-to-market value posited by Fama and French. Carhart's addition of the momentum factor was inspired by the empirical finding that stocks that have performed well in the past tend to continue to perform well in the near future, and conversely, stocks that have performed poorly tend to continue their underperformance.

The Carhart Four-Factor Model is expressed through the equation:

$$R_{i,t} - R_{f,t} = \alpha + \beta_m (MKT_t - R_{f,t}) + \beta_s SMB_t + \beta_v HML_t + \beta_m MOM_t + \epsilon_{i,t}$$

Where:

- $R_{i,t} - R_{f,t}$ is the excess return of the stock,
- α is the intercept,
- $MKT_t - R_{f,t}$ represents the market excess return,
- SMB_t is the size premium,
- HML_t is the value premium,
- MOM_t denotes the momentum factor, and
- $\epsilon_{i,t}$ is the error term.

The model effectively integrates momentum into the asset pricing framework, providing investors and researchers with a more nuanced tool for explaining and predicting stock returns. The inclusion of momentum, a factor based on past return trends, underscores the importance of historical performance patterns in assessing future returns, adding a temporal dimension to the risk factors previously identified by Fama and French.

2.9 Fama and French Five Factor Model

Eugene F. Fama and Kenneth R. French, further building on the insights provided by their Three Factor Model, introduced two additional factors to better encapsulate the complexities inherent in stock returns. This expansion led to the creation of the Fama and French Five Factor Model, which incorporates profitability (RMW) and investment (CMA) alongside the original factors of market risk, size, and value (Fama & French, 2015). The model is formalized as follows:

$$R_{i,t} - R_{f,t} = \alpha + \beta_m (MKT_t - R_{f,t}) + \beta_s SMB_t + \beta_v HML_t + \beta_r RMW_t + \beta_c CMA_t + \epsilon_{i,t}$$

Where:

- $R_{i,t} - R_{f,t}$ represents the excess return of asset i over the risk-free rate at time t,
- α is the intercept, capturing the average excess return unexplained by the model,
- β_m, β_s, β_v, β_r, and β_c are the sensitivities of the asset returns to the market excess return (MKT), size premium (SMB), value premium (HML), profitability (RMW), and investment (CMA) factors, respectively,
- $MKT_t - R_{f,t}$ is the market excess return over the risk-free rate,
- SMB_t, HML_t, RMW_t, and CMA_t are the size, value, profitability, and investment factors at time t,
- $\epsilon_{i,t}$ is the error term for asset i at time t, capturing the return variance not explained by the five factors.

The inclusion of RMW (Robust Minus Weak) reflects the performance differential between companies with high and low profitability, while CMA (Conservative Minus Aggressive) captures the return variance between firms that invest conservatively and those that invest aggressively. This model's development was underpinned by the empirical observation that firms exhibiting high profitability and conservative investment strategies frequently outperform their counterparts, thereby offering a refined lens through which to view the determinants of equity returns.

The Five Factor Model has had a significant impact on both the academic and practical realms of finance, illustrating the nuanced relationship between a company's financial strategies and its valuation in the marketplace. By providing a comprehensive framework that accounts for a broader spectrum of risk factors influencing stock performance, the model enhances our understanding of asset pricing and portfolio management, setting a new benchmark for research and investment strategy development in the field.

2.10 Model Preference Discussion

The evolution of asset pricing models has been marked by a continuous quest to better explain the variations in stock returns beyond what is accounted for by market indices alone. The Capital Asset Pricing Model (CAPM), introduced by William F. Sharpe (1964), provided the foundational framework, asserting that the expected return of a security or portfolio is determined by its beta, reflecting its market risk exposure. Despite CAPM's widespread adoption and its pivotal role in the development of financial theory, its limitations became apparent as it failed to account for various anomalies observed in asset pricing.

In response to these shortcomings, Eugene F. Fama and Kenneth R. French developed the Three-Factor Model (1993), introducing size (SMB) and value (HML) factors alongside the market risk factor to provide a more comprehensive explanation of stock returns. This model significantly advanced the understanding of asset pricing by accounting for the empirical anomalies that CAPM could not explain.

Building on the Three-Factor Model, Mark Carhart proposed the Four-Factor Model (1997) by adding momentum (MOM) as a fourth factor, recognizing the empirically observed tendency of stocks to continue moving in their recent direction, whether upward or downward. Carhart's inclusion of momentum acknowledged the importance of historical performance patterns in predicting future returns, enhancing the model's explanatory power.

Fama and French further refined their approach by introducing the Five-Factor Model (2015), which added profitability (RMW) and investment (CMA) factors to the market risk, size, and value factors. This model aimed to capture the effects of a company's profitability and investment behaviors on stock performance, offering a more nuanced understanding of the drivers of equity returns.

The expansion of the Fama and French model from three to five factors has sparked considerable debate within the academic and investment communities regarding the optimal number of factors needed to accurately capture stock return variances. Critics of the expanded models argue that the addition of more factors risks overfitting and reduces the models' practical applicability. Proponents, however, contend that the additional factors address significant anomalies unexplained by CAPM and the Three-Factor Model, providing a more comprehensive view of market dynamics.

Given the vibrant discourse surrounding the optimal composition of factor models for elucidating stock returns, this study opts to employ the Fama and French Five Factor Model. This decision is rooted in the model's comprehensive approach, incorporating market risk, size, value, profitability, and investment factors—providing a nuanced framework for understanding the multifaceted influences on stock performance. The Five Factor Model's inclusion of profitability (RMW) and investment (CMA) factors is particularly compelling, as it aligns with the study's objective to delve into the intrinsic characteristics of firms that contribute to their market valuation and returns.

This choice is supported by the recognition that while momentum, as introduced by the Carhart Four-Factor Model, offers valuable insights into price trends, the broader and more granular examination afforded by the Five Factor Model is essential for capturing the complexities of equity returns. By integrating a wider spectrum of risk factors, the Five Factor Model facilitates a more detailed exploration of how a company's financial strategies impact its stock performance, thereby providing a solid foundation for this study's analysis.

In conclusion, after considering the merits and limitations of various factor models, the study has chosen to utilize the Fama and French Five Factor Model. This selection is guided by the model's ability to offer a comprehensive understanding of the determinants of stock returns, ensuring a robust analytical framework that is well-suited to the research questions at hand. The ongoing debate within the academic and investment communities underscores the evolving nature of financial theory, and the adoption of the Five Factor Model reflects a commitment to leveraging advanced methodologies that best capture the complexities of market behavior.

2.11 Understanding Risk-Adjusted Returns

Modern financial theory underscores the necessity of evaluating investment performance through risk-adjusted metrics, extending the analysis beyond traditional models. This section introduces key metrics such as the Sharpe Ratio, Sortino Ratio, volatility, and correlation coefficient, highlighting their importance in a comprehensive financial analysis. Their relevance to investment strategies, notably the Magic Formula, illustrates the critical role of risk considerations in formulating strategies aimed at optimizing investment performance.

2.11.1 The Sharpe Ratio

Developed by William F. Sharpe (1964), the Sharpe Ratio is a critical measure for evaluating the risk-adjusted return of an investment portfolio. It quantifies the additional return an investor receives per unit of increase in risk, specifically comparing the portfolio's excess return over the risk-free rate to its standard deviation. The formula for the Sharpe Ratio is

$$\frac{R_p - R_f}{\sigma_p}$$

where R_p represents the portfolio’s historical return, R_f denotes the risk-free rate of return, and σ_p is the standard deviation of the portfolio's excess returns. This measure is invaluable for assessing the efficiency of various investment strategies, providing insights into their ability to generate returns above the risk-free rate while accounting for the volatility of those returns. By evaluating investment strategies through the lens of the Sharpe Ratio, investors and analysts can better understand the trade-offs between risk and return, making it a fundamental component of modern portfolio theory and investment analysis.

2.11.2 The Sortino Ratio

The Sortino Ratio, a modification of the Sharpe Ratio, was developed to focus solely on downside risk, offering a more targeted measure of risk-adjusted performance (Sortino & van der Meer, 1991). Unlike the Sharpe Ratio, which considers the total standard deviation of portfolio returns, the Sortino Ratio only considers the volatility of negative asset returns. This distinction is crucial for investors who are particularly concerned with downside risk rather than overall volatility. The Sortino Ratio formula is

$$\frac{R_p - R_f}{\sigma_d}$$

where R_p represents the portfolio's return, R_f denotes the risk-free rate of return, and σ_d is the standard deviation of the portfolio's negative returns. By focusing on the downside deviation, the Sortino Ratio provides a more refined assessment of the risk involved in achieving excess returns over the risk-free rate, particularly in investment strategies where the management of downside risk is a priority. This metric is especially relevant in the evaluation of investment strategies that aim to minimize losses during market downturns, reinforcing the importance of considering both the magnitude and direction of risk when analyzing investment performance.

2.11.3 Volatility as a Measure of Risk

Volatility, often quantified as the annualized standard deviation of returns, gauges the degree of variability in an investment's returns over time, serving as a key indicator of the associated risk. High volatility denotes substantial price fluctuations, implying increased risk, while low volatility indicates steadier, more predictable returns. This principle is pivotal in assessing investment strategies such as Joel Greenblatt's Magic Formula, aimed at identifying undervalued, high-quality stocks. Despite their potential for short-term volatility, these stocks are anticipated to deliver superior long-term returns. Understanding the implications of volatility on investment risk and return profiles has been a cornerstone of modern portfolio theory since the seminal work of Markowitz (1952), laying the groundwork for risk management and diversification strategies that remain central to financial decision-making today.

2.11.4 Correlation Coefficient in Investment Analysis

The correlation coefficient is pivotal in evaluating the relationship between the performance of an investment strategy and broader market movements. A positive correlation indicates that the investment strategy and market returns move in the same direction, while a negative correlation suggests inverse movement. For investors applying Joel Greenblatt's Magic Formula, assessing the correlation coefficient between the formula's returns and market performance is crucial. It allows an understanding of how closely the strategy mirrors or deviates from overall market trends, shedding light on its diversification benefits and market sensitivity. This analysis is essential for identifying strategies that not only aim for high returns but also enhance portfolio diversification, potentially mitigating risk in varied market conditions.

3 Data and Research Methodology

This chapter outlines the methodology employed in this thesis, detailing the processes of data collection and processing, the formulation of hypotheses, the application of Joel Greenblatt’s Magic Formula, the construction of the Fama and French Five Factor Model, and the statistical methods used for data analysis. This structured approach ensures the rigor and reproducibility of the research findings.

3.1 Data Collection and Processing

This research utilizes the Sharadar Core US Equities Bundle, accessed through Nasdaq Data Link (Nasdaq Data Link, 2023), offering a comprehensive dataset filled with financial and operational metrics for a wide array of companies listed on the US stock market over a 25-year period, from 1998 to 2023. The dataset is instrumental in examining the effectiveness of Joel Greenblatt's Magic Formula and its performance relative to the Fama and French Five Factor Model, focusing on various variables such as earnings, book values, market capitalizations, stock returns, and specific investment and profitability metrics.

The study examines multiple market capitalization segments—ranging from the top 3500, 2500, 1500, to 500 companies—to conduct a detailed analysis of how different tiers of market capitalization respond to the Magic Formula strategy within the context provided by the Five Factor Model. This stratified approach allows for an insightful exploration into the diverse impacts of market capitalization on investment strategy performance.

During the data processing phase, extensive cleaning and preparation were undertaken to maintain the dataset's integrity and reliability. This involved the removal of companies lacking the necessary accounting data for calculating the two critical factors of the Magic Formula in any given quarter, ensuring that only complete and accurate data were used in the analysis.

Crucially, the Sharadar database is distinguished by its provision of point-in-time data, a key feature that ensures the fidelity of backtesting by eliminating the risk of look-ahead bias (Nasdaq Data Link, 2023). This attribute is essential for conducting historical performance analyses, as it guarantees that the data reflects only the information that would have been available to investors at the time, thus upholding the integrity of the backtesting and safeguard against overfitting. The database's comprehensive coverage, data quality, and adherence to point-in-time reporting collectively provide a robust foundation for the empirical investigation. These qualities support an in-depth exploration of value investing principles and factor-based market analysis, facilitating a thorough evaluation of the Magic Formula's performance amidst evolving market dynamics and risk factors.

3.2 Hypotheses Formulation

In alignment with the study's theoretical underpinnings and initial empirical observations, two primary hypotheses were posited:

- **H1:** The Magic Formula demonstrates significant performance variation across the top 3500 and top 500 market capitalization segments, which can be attributed to its ability to generate alpha after accounting for risk factors in the Fama and French Five Factor Model.

- **H2:** The Fama and French Five Factors provide a substantial explanatory power for the returns achieved by the Magic Formula, indicating the influence of size, value, profitability, and investment factors on market efficiency and the identification of risk premiums.

3.3 Application of the Magic Formula

The Magic Formula, a value investing strategy delineated by Joel Greenblatt, is operationalized in this study using financial data accessible through NASDAQ Data Link. This platform provides a robust dataset that is processed using Python to compute two pivotal financial ratios critical to the Magic Formula's approach.

The first component of the formula is the Return on Capital (ROC), defined as

$$\frac{EBIT}{Net\ Working\ Capital + Net\ Fixed\ Assets}$$

This calculation measures the efficiency of a company in generating pre-tax earnings (EBIT) from its employed capital, which includes both working capital and fixed assets.

The second component is the Earnings Yield (EY), expressed as

$$\frac{EBIT}{Enterprise\ Value}$$

Here, the Enterprise Value is the sum of the market value of equity and net interest-bearing debt, providing a comprehensive measure of a company's valuation relative to its pre-tax earnings (EBIT).

Upon calculating these ratios, we establish a composite ranking that serves as the basis for selecting investments. Companies are then classified into deciles based on their combined scores. The first decile includes the top 10% of companies, which are predicted to perform exceptionally well in the upcoming period. This decile-based system enables a structured method for identifying promising investment opportunities, aligning with Greenblatt’s strategic emphasis on investing in high-quality, undervalued companies.

This ranking system is updated quarterly to incorporate the most recent financial data and market valuations, ensuring the investment strategy adapts to changing economic landscapes. This dynamic method of portfolio management is essential to the Magic Formula's effectiveness, allowing it to exploit the disparities between market price and intrinsic value effectively.

3.4 Construction of the Fama and French Five Factor Model

The Fama and French Five Factor Model is pivotal for dissecting the complexities of stock returns, incorporating five distinct factors: excess market return (EMR), size (SMB), value (HML), profitability (RMW), and investment (CMA). The methodology for calculating each factor is detailed below, with a focus on HML, RMW, and CMA based on the distribution of the sample into the top and bottom 30% brackets.

Excess Market Return (EMR): EMR is calculated as the difference between the broad market portfolio return and the risk-free rate, capturing the market's overall risk premium.

Size (SMB - Small Minus Big): SMB is determined by comparing the average returns of small-cap stocks with those of large-cap stocks. Stocks are categorized into small and big groups based on their median market capitalization, and SMB is calculated as the mean return of small stocks minus the mean return of big stocks.

Value (HML - High Minus Low): HML measures the premium of value stocks over growth stocks. Stocks are classified into high, middle, and low groups based on their book-to-market (BM) ratios. HML is calculated as the mean return of the top 30% of stocks (high BM) minus the bottom 30% (low BM).

Profitability (RMW - Robust Minus Weak): RMW evaluates the premium associated with companies that exhibit robust profitability compared to those with weak profitability. Profitability is determined by earnings before interest and taxes (EBIT) divided by book equity. RMW is calculated as the mean return of the top 30% profitability stocks minus the bottom 30%.

Investment (CMA - Conservative Minus Aggressive): CMA contrasts the performance of firms that invest conservatively against those that invest aggressively. The investment rate is assessed by the change in total assets. CMA is the mean return of the bottom 30% of firms by asset growth (conservative) minus the top 30% (aggressive).

These factor calculations are performed quarterly using data from the Sharadar Core US Equities Bundle (Nasdaq Data Link, 2023). This regular and meticulous approach facilitates a dynamic analysis of how these factors impact stock returns and the effectiveness of investment strategies like the Magic Formula in varying market conditions. By quantifying the premiums associated with size, value, profitability, and investment, this model offers a comprehensive framework for understanding risk-adjusted returns and market anomalies.

3.5 Quantitative Analysis Methods

This study employs multiple regression models within a quantitative framework to examine the relationships between the returns of portfolios derived from Joel Greenblatt's Magic Formula and the risk factors identified in the Fama and French Five-Factor Model. These factors include market premium, size premium, value premium, profitability, and investment. The analysis aims to assess the explanatory power of these factors in accounting for the variability in returns produced by the Magic Formula strategy.

The regression models are constructed using the *statsmodels* library in Python, selected for its robust capabilities in statistical analysis, enabling precise control over model specifications and detailed diagnostic testing. The models are carefully designed to isolate and quantify the impact of each risk factor outlined by Fama and French on the performance of portfolios ranked according to the Magic Formula, facilitating a comprehensive understanding of the factors influencing portfolio returns.

To ascertain the robustness and validity of the regression analysis, the study incorporates a series of diagnostic tests addressing potential statistical issues, including multicollinearity, heteroscedasticity, autocorrelation, and the normality of residuals. These tests are integral to confirming the assumptions underpinning the Ordinary Least Squares methodology utilized in the analysis.

The Variance Inflation Factor is employed to evaluate multicollinearity among the independent variables, ensuring no single variable unduly influences the regression outcomes. Acceptable VIF values below 10 confirm the absence of problematic multicollinearity. The Breusch-Pagan test is used to assess heteroscedasticity, verifying the uniformity of variance across the dataset's residuals.

Autocorrelation is assessed using the Durbin-Watson statistic, which helps identify any correlation between sequential residuals that might compromise the integrity of the regression results. Additionally, the normality of residuals is evaluated through the Shapiro-Wilk, Jarque-Bera, and Omnibus tests, complemented by visual inspection using Q-Q plots. These measures ensure that the residuals adhere closely to a normal distribution, a critical assumption for the validity of the regression outputs.

Through these comprehensive diagnostic tests, the study ensures that the regression models are robust and valid, providing reliable insights into the economic forces affecting investment returns.

4 Empirical Analysis

In this chapter, we present a detailed empirical analysis of portfolios constructed using Joel Greenblatt's Magic Formula, focusing on their performance outcomes and factor influences between the largest 3500, 2500, 1500 and 500 companies by market capitalization segments of the US stock market from 1998 to 2023. This analysis employs regression techniques to understand the influence of market factors as defined by the Fama and French Five Factor Model on the portfolios' returns. The outcomes of this analysis are encapsulated in a comparative table that highlights the key metrics and findings, providing a clear overview of how these factors impact the performance of the Magic Formula portfolios.

4.1 Comparative Performance Analysis

In examining the dataset across different market capitalization tiers, descriptive statistics reveal significant variances in financial metrics. The analysis highlights the distribution, variability, and the financial health of companies across the spectrum, from the broadest (top 3500) to a more concentrated subset (top 500). This section delineates the mean, median, and standard deviation values for earnings yield, return on capital, market capitalization, alongside key factors from the Fama and French model, offering insights into the underlying financial landscape these companies navigate.

The table below presents the results from performance and regression analysis on the top 3500, top 2500, top 1500, and top 500 companies sorted by market capitalization, based on quarterly underlying data. This comparison allows us to discern the variations in portfolio performance, factor significance, and coefficient changes across these market segments.

Table 4.1: Comparative Analysis of Magic Formula Portfolio and Factor Coefficients Across Different Market Capitalization Segments (1998-2023)

Statistics (1998-2023, quarterly data)	**Top 3500**	**Top 2500**	**Top 1500**	**Top 500**
Annualized Magic Formula Portfolio Return	20.7%	17.0%	12.6%	11.3%
Annualized Market Return	10.7%	9.9%	8.2%	6.9%
Excess Return	10.0%	7.1%	4.4%	4.4%
R-squared	0.949	0.937	0.934	0.863
Intercept	**0.015****	**0.012***	**0.009****	**0.011****
EMR Coef	**0.939****	**0.923****	**0.913****	**0.881****
SMB Coef	**0.222****	**0.165***	-0.029	-0.149
HML Coef	0.068	0.023	**0.128****	**0.211****
RMW Coef	**0.167***	**0.204****	**0.214****	**0.249****
CMA Coef	0.054	-0.009	-0.107	**-0.154****
Annualized Portfolio Std Dev	0.326	0.231	0.168	0.142
Annualized Market Std Dev	0.294	0.256	0.206	0.176
Annualized Portfolio Volatility	0.235	0.198	0.158	0.135
Annualized Market Volatility	0.236	0.219	0.184	0.153
Annualized Portfolio Sharpe Ratio	0.287	0.264	0.234	0.220
Annualized Market Sharpe Ratio	0.137	0.127	0.109	0.082
Annualized Excess Sharpe Ratio	0.150	0.137	0.125	0.138
Correlation Coefficient (σ_p, σ_m)	0.961	0.957	0.929	0.841
Quarterly Portfolio Sortino Ratio	0.372	0.282	0.161	0.146
Quarterly Market Sortino Ratio	0.107	0.081	0.037	0.003
Annualized Best Portfolio Return	140%	99.0%	71.6%	57.3%
Annualized Max Portfolio Drawdown	32.8%	37.5%	38.3%	31.0%

* Indicates significance at the 5% level.

** Indicates significance at the 1% level.

The latter sections of this chapter are dedicated to presenting detailed visual analyses for the top 3500 and top 500 market capitalization segments. This focus is justified by distinct quantitative findings from the comparative analysis of these segments relative to the top 2500 and top 1500 market capitalization segments throughout the 1998-2023 study period.

One critical metric under review is the Portfolio Quarterly Max Drawdown, which reflects the most significant quarterly performance decline. The top 3500 and top 500 segments exhibited Max Drawdowns of 32.8% and 31.0%, respectively, suggesting a more contained risk profile compared to the top 2500 and top 1500 segments, which recorded higher max drawdowns of 37.5% and 38.3%. This difference highlights a more pronounced susceptibility to sharp quarterly declines in the latter segments.

Additionally, the analysis explores the Excess Annualized Sharpe Ratios, a measure of risk-adjusted performance that calculates the additional return per unit of risk over the market benchmark. Here, the top 3500 and top 500 segments outperform with Excess Annualized Sharpe Ratios of 0.150 and 0.138, respectively, indicating superior risk-adjusted returns compared to the 0.137 and 0.125 ratios of the top 2500 and top 1500 segments. This suggests that the selected segments not only demonstrate lower risk but also convert this advantage into more favorable risk-adjusted returns.

The ensuing empirical analysis will thus shed light on the performance and risk management efficiency of the top 3500 and top 500 segments, aiming to offer insights into their stability and effectiveness across different market conditions.

For detailed visual representations of the top 2500 and top 1500 segments, readers are referred to the appendix.

4.2 Performance Analysis of Magic Formula Portfolios

This section elucidates the differential performance of the Magic Formula across the top 3500 versus the top 500 companies. It details the annualized returns for portfolios derived from the top decile of companies as identified by the formula, contrasting these with the broader market's performance. Specifically, the analysis investigates the consistency and magnitude of the formula's outperformance across both subsets, providing a nuanced understanding of its effectiveness in varying market cap contexts.

4.2.1 Annualized Returns Across Market Cap Segments

The Magic Formula, conceptualized by Joel Greenblatt, is designed to pinpoint high-quality companies at undervalued prices. This section examines the annualized returns of Magic Formula portfolios across two key market capitalization segments, the top 3500 and top 500, over the period from 1998 to 2023, and contextualizes these returns against broader market performance.

The top 3500 segment, encompassing a broad portion of the market, achieved an impressive, annualized return of 20.7%. This figure is nearly double the market return of 10.7% for the same period, resulting in an excess return of 10.0% — a return that is 93% higher, as shown in Figure 4.1. Such a substantial outperformance underscores the Magic Formula's efficacy in large, diversified segments of the market, where its criteria for selecting companies based on earnings yield and return on capital can identify significant opportunities for growth.

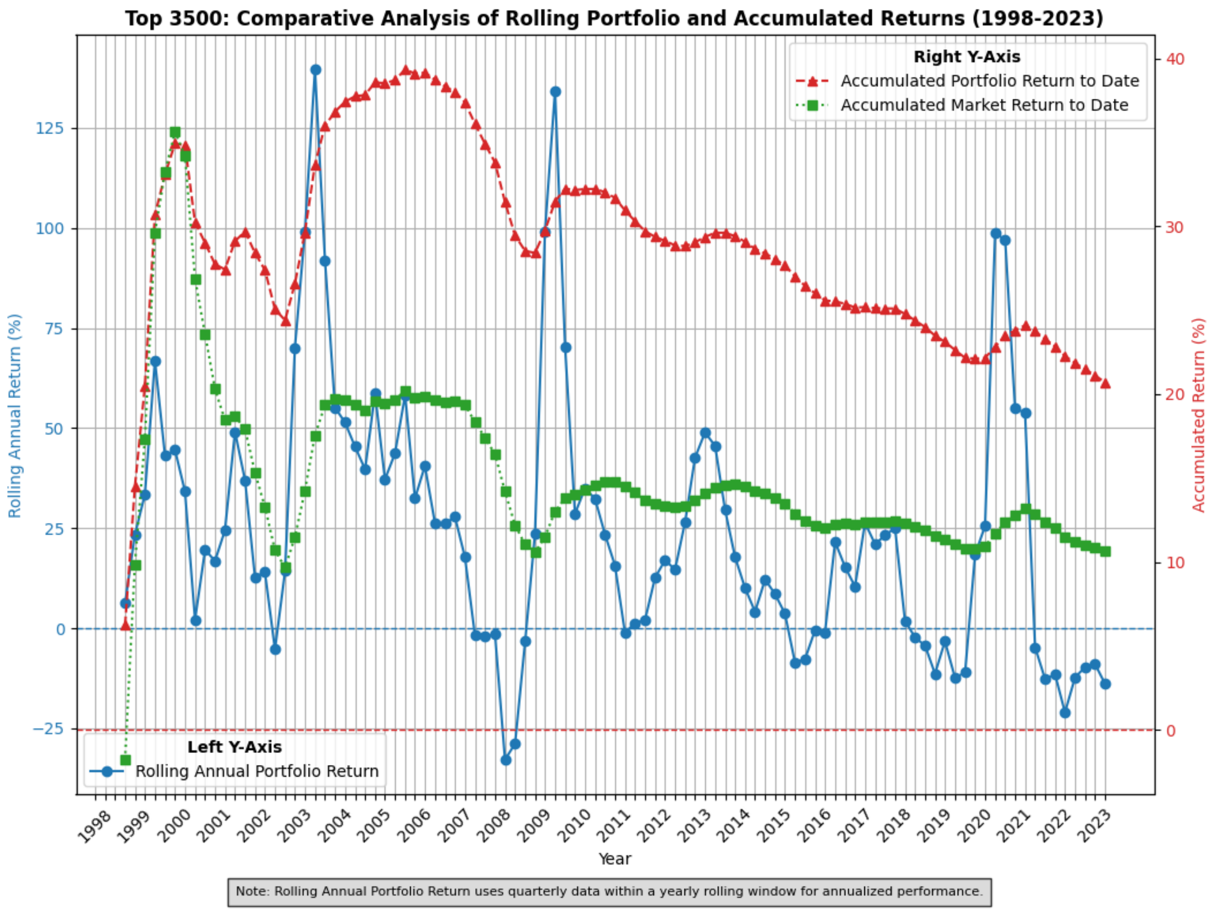

Figure 4.1: Comparative Analysis of Rolling Annual Portfolio Returns (left y-axis) shown with Accumulated Portfolio and Market Returns (right y-axis) for Top 3500 companies (1998-2023)

In contrast, the top 500 segment, which focuses on larger cap companies, recorded an annualized return of 11.3%, above the market return of 6.9% for comparable companies, yielding an excess return of 4.4% — a return that is 64% higher, as shown in Figure 4.2. While the absolute returns are lower than those of the top 3500 segment, the performance of the top 500 is notable for its ability to generate substantial excess returns over the market benchmark, demonstrating the Magic Formula's versatility and effectiveness even when applied to more established, higher market cap companies.

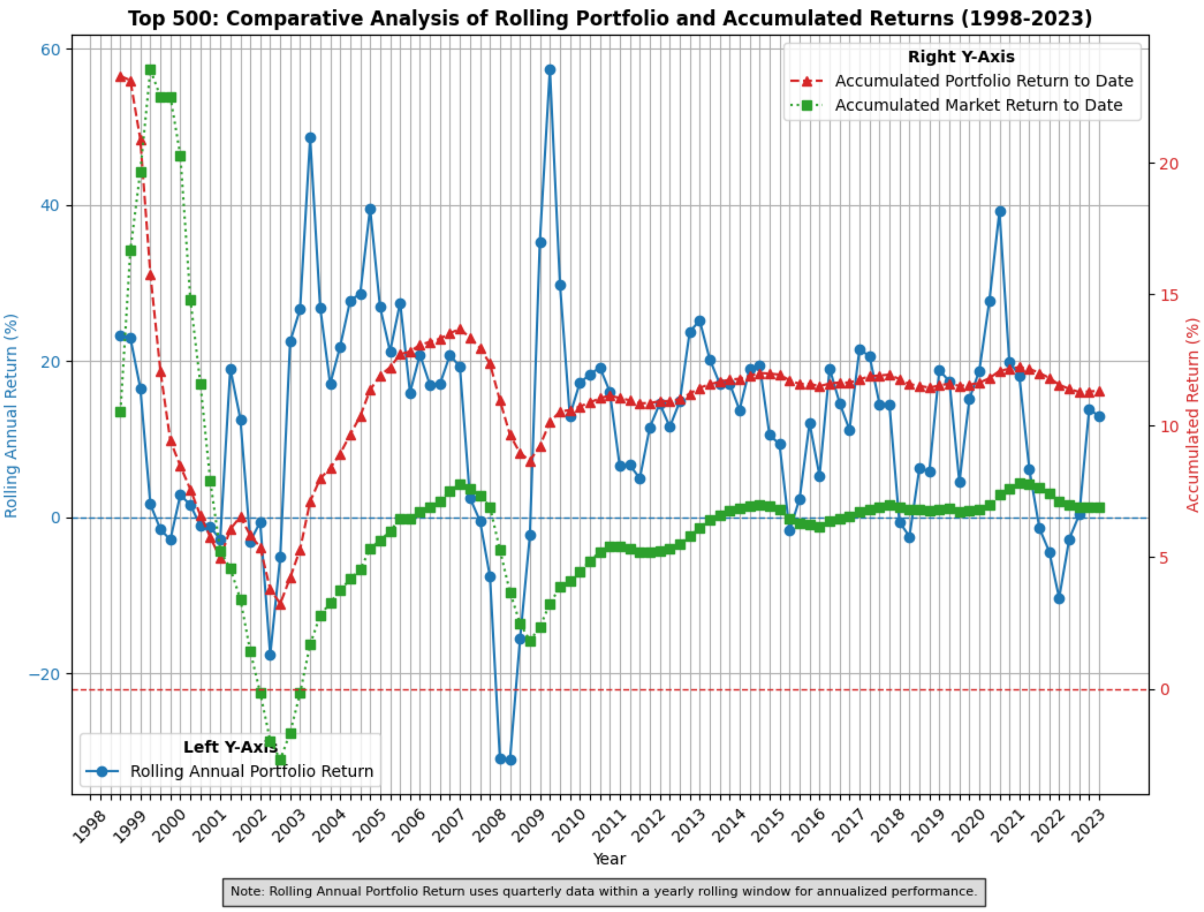

Figure 4.2: Yearly Portfolio Returns (left y-axis) shown with Portfolio and Market Annualized Returns (right y-axis) for Top 500 companies in the years 1998-2023

The differential in performance between the top 3500 and top 500 segments highlights the impact of market capitalization on the Magic Formula's success. The broader top 3500 segment benefits from a wider selection of companies, potentially capturing more undervalued opportunities across various industries and sectors. Meanwhile, the top 500 segment's solid performance affirms the Magic Formula's capacity to discern value even among the largest and often closely watched companies in the market.

Overall, the annualized returns and excess returns for both the top 3500 and top 500 segments illustrate the Magic Formula's potent application across different market cap segments. Whether applied to a wide array of companies or focused on the large-cap end of the market, the strategy consistently delivers returns that surpass market averages, showcasing its relevance and adaptability to diverse investment landscapes.

4.2.2 Risk-Adjusted Performance Evaluation

The assessment of investment performance is incomplete without considering the risks undertaken to achieve returns. This section delves into the risk-adjusted performance of Joel Greenblatt's Magic Formula, employing two critical metrics: the Sharpe and Sortino ratios. These ratios provide a nuanced view of the strategy's efficiency by measuring excess returns per unit of total and downside risk, respectively. By analyzing these metrics for the top 3500 and top 500 market cap segments from 1998 to 2023, we aim to uncover the inherent risk-return characteristics of the Magic Formula and its capacity to outperform market benchmarks within these segments.

Sharpe Ratio

In the realm of finance, the Sharpe ratio stands as a pivotal metric for assessing risk-adjusted returns, enabling investors to understand the excess return obtained per unit of risk. This evaluation focuses on elucidating the risk-adjusted performance of the Magic Formula strategy within the top 3500 and top 500 market cap segments, over the period from 1998 to 2023, by comparing their Portfolio Annualized Sharpe Ratios with the respective Market Annualized Sharpe Ratios.

For the top 3500 segment, the analysis reveals a Portfolio Annualized Sharpe Ratio of 0.287, contrasted with a Market Annualized Sharpe Ratio of 0.137. Development of the Annualized Sharpe Ratios for top 3500 throughout the 25 years can be seen in Figure 4.3. Risk-Free Rate has been included to show inverse correlation. This Sharpe Ratio disparity suggests that the Magic Formula strategy, when applied to a broad spectrum of the market, not only achieves higher returns relative to the risk-free rate but does so with a commendable efficiency of risk utilization. Such a finding supports the hypothesis that diversified exposure across a vast array of companies can mitigate unsystematic risk while harnessing the potential for value identification across sectors.

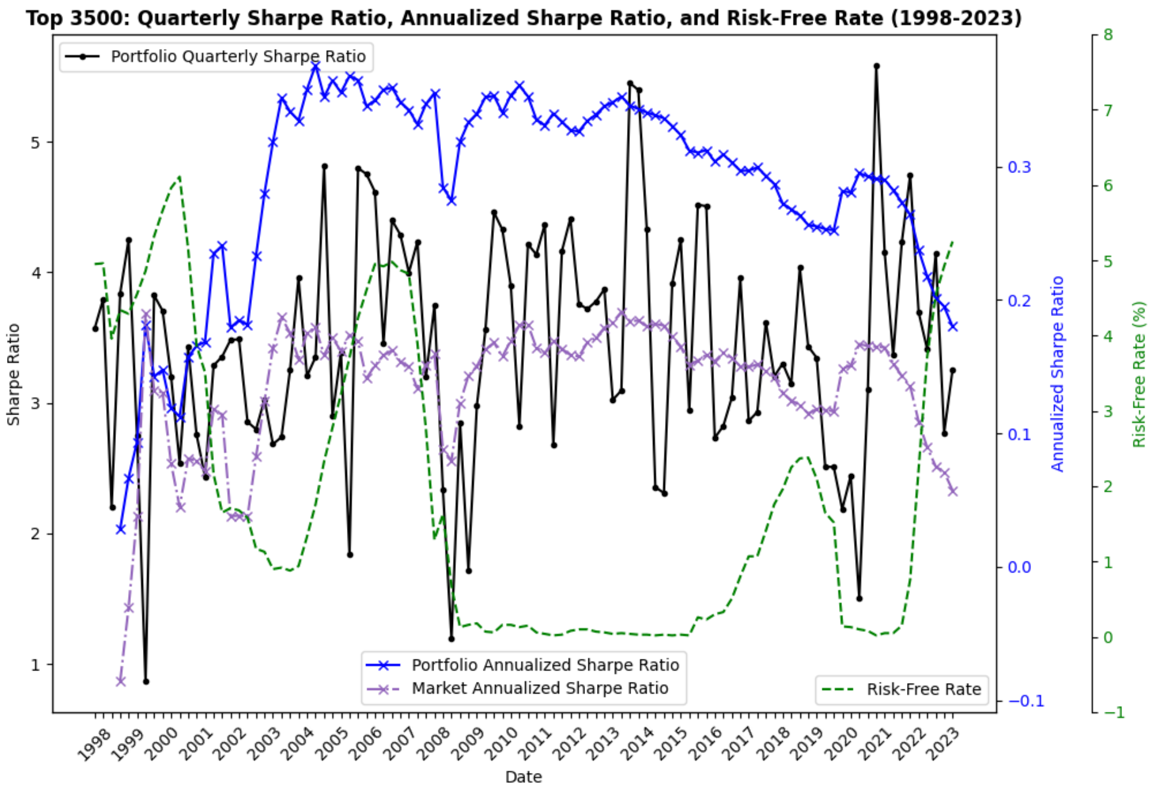

Figure 4.3: Quarterly Sharpe Ratio (left y-axis) shown with Portfolio and Market Annualized Sharpe Ratio (right y-axis) for Top 3500 companies (1998-2023)

Conversely, the top 500 segment, which encompasses larger capitalization stocks, presents a Portfolio Annualized Sharpe Ratio of 0.220 against a Market Annualized Sharpe Ratio of 0.082. Development of the Annualized Sharpe Ratios throughout the 25 years can be seen in Figure 4.4. Risk-Free Rate has been included to show inverse correlation. Despite the inherently lower volatility associated with large-cap stocks, this segment's Sharpe Ratio indicates a successful application of the Magic Formula in extracting value and delivering risk-adjusted returns that significantly exceed the market average. This outcome serves to underline the strategy's effectiveness, even within a universe of companies typically characterized by lower return volatility.

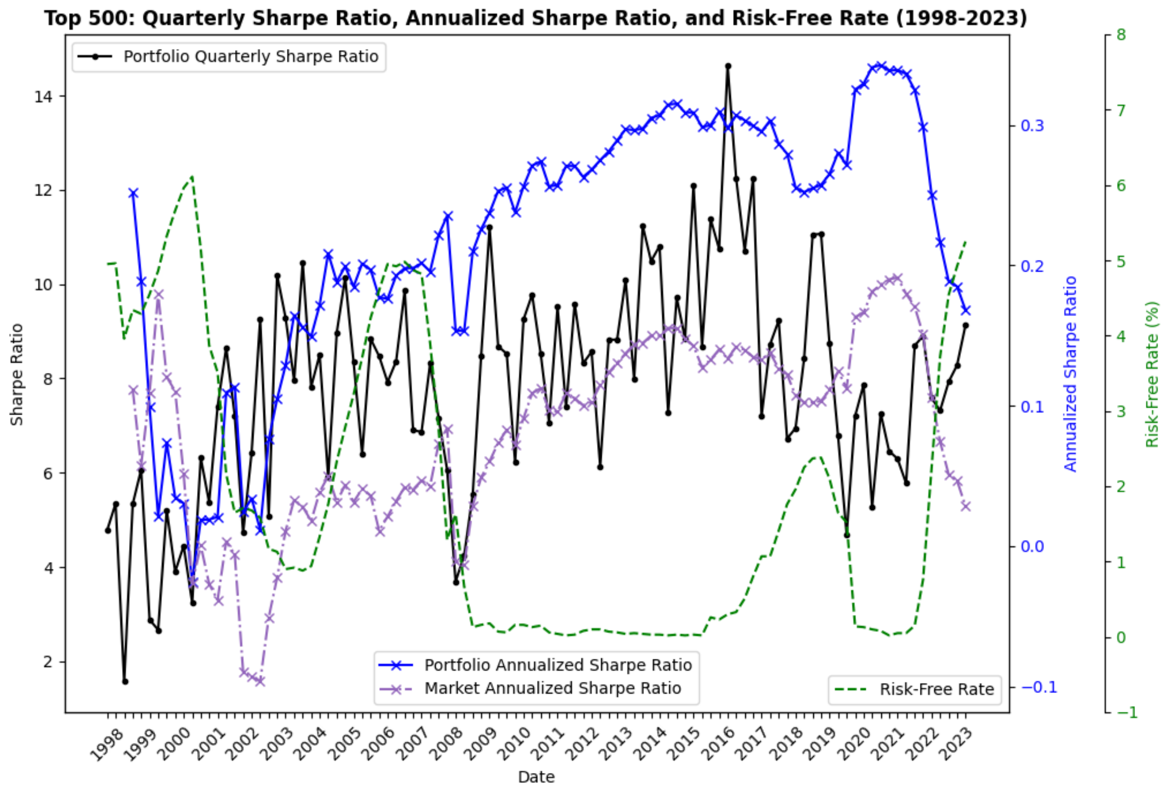

Figure 4.4: Quarterly Sharpe Ratio (left y-axis) shown with Portfolio and Market Annualized Sharpe Ratio (right y-axis) for Top 500 companies (1998-2023)

Delving into the Excess Annualized Sharpe Ratios, the top 3500 and top 500 segments register values of 0.150 and 0.138, respectively. These metrics are emblematic of the strategy's capacity to offer returns that compensate for the assumed risk, above and beyond the market benchmark. Notably, the top 3500 segment exhibited peaks above 0.3 during the years 2003 to 2016, suggesting a period of outperformance that aligns with historic market rallies and perhaps a broader capture of market upswings due to its diversified nature. In contrast, the top 500 segment, which may reflect a more concentrated selection of large-cap entities, experienced its significant peaks later, specifically from 2014 to 2016, and again in 2020-2021. This temporal variance in peak performance between the two segments underscores the influence of market dynamics and economic cycles on investment strategies, and possibly, a shift in investor sentiment towards large-cap or sector-specific investments during those later periods.

The excess ratios affirm that the higher returns provided by the Magic Formula are not merely a function of increased risk exposure but are indicative of strategic selection that prioritizes value and quality. The differential timing of the Sharpe ratio peaks further illustrates that the Magic Formula's principles of selecting undervalued stocks with a high return on capital are not uniformly effective across all market conditions but may have periods of pronounced efficacy.

Sortino Ratio

To augment the risk-adjusted performance analysis, this study also incorporates the Sortino ratio, which differentiates itself from the Sharpe ratio by focusing solely on downside risk. This metric is particularly useful for investors who are more concerned with the potential for losses than the overall volatility. The Sortino ratio calculation for the top 3500 and top 500 segments underscores the Magic Formula's ability to minimize the impact of negative returns on the overall portfolio performance.

The top 3500 segment exhibits a Quarterly Portfolio Sortino Ratio of 0.372, significantly higher than the Market's 0.107, highlighting the Magic Formula's adeptness at navigating market downturns with minimal detrimental effect on the portfolio. Similarly, in the top 500 segment, the Portfolio Quarterly Sortino Ratio of 0.146 surpasses the Market's 0.003, further evidencing the strategy's resilience against adverse market conditions.

In summary, the investigation into Sharpe and Sortino ratios for the top 3500 and top 500 market cap segments underscores the efficacy of Joel Greenblatt's Magic Formula in achieving superior risk-adjusted returns. By simultaneously addressing total and downside risks, this analysis affirms the strategy's robustness in navigating market inefficiencies across diverse market segments. Such findings emphasize the strategic value of a comprehensive risk assessment in the realm of investment strategy, validating the Magic Formula's foundational principle of selecting undervalued, high-return companies. Furthermore, the consistent performance of the Magic Formula across varying market conditions attests to the enduring relevance of fundamental analysis in portfolio management, highlighting its potential to yield favorable outcomes over a prolonged period without necessitating adjustments to its core methodology.

4.3 Factor Analysis and Portfolio Performance

This section delves into the performance of portfolios crafted using Joel Greenblatt's Magic Formula against the backdrop of the Fama and French Five-Factor Model, concentrating on the largest 3500 and 500 companies by market capitalization. The model evaluates five critical dimensions of market behavior: excess market return (EMR), size (SMB), value (HML), profitability (RMW), and investment (CMA) factors, providing a comprehensive framework for analyzing risk and return characteristics.

4.3.1 Exposure to Fama and French Five Factors

An analysis of the Magic Formula portfolios' alignment with the Excess Market Return (EMR) factor reveals significant positive coefficients in both the top 3500 (0.939) and top 500 (0.881) segments. This alignment indicates a robust linkage with overarching market trends, suggesting that the portfolios are highly responsive to general market movements. The notable significance of these coefficients, established at the 1% level, underscores the portfolios' integration with the market's overall direction, yet highlights the unique stock selection methodology underpinning the Magic Formula's performance ethos.

4.3.2 Size and Value Factors

The Size (SMB) and Value (HML) factors play pivotal roles in the Fama and French framework, with their influence varying across different market capitalization segments. The top 3500 segment demonstrates a significant SMB coefficient (0.222) at the 1% level, reinforcing the strategy's effectiveness in leveraging the size anomaly — smaller companies often yield higher expected returns due to their risk-return profile. This aligns with the Magic Formula's approach, which appears to favor these smaller, potentially undervalued companies.

Contrastingly, the HML coefficient for the top 3500 segment does not reach conventional levels of statistical significance (0.068), suggesting that the classic value premium — where high book-to-market stocks outperform low — is not a primary driver in this segment for the period studied. This may imply a more complex interaction between the Magic Formula's stock selection criteria and the traditional value factor, or it may suggest that other factors play a more dominant role in driving returns in this broader market segment.

In the more concentrated top 500 segment, the negative SMB coefficient (-0.149) is consistent with the segment's focus on larger companies, where the size effect is naturally diminished. However, the significantly positive HML coefficient (0.211) at the 1% level indicates a substantial value tilt. This underlines the Magic Formula's capacity to identify undervalued large-cap stocks that offer a value premium, even in a segment where market efficiency is generally assumed to be higher.

The divergent significance and direction of the SMB and HML coefficients between the top 3500 and top 500 segments illustrate the variable nature of factor impacts across different scopes of market capitalization. In the broader top 3500 segment, the Magic Formula's performance seems more connected with capturing the size premium, while in the top 500, value investing principles, as captured by the HML factor, are more pronounced. This provides a multifaceted view of the Magic Formula's adaptability and the importance of factor consideration in portfolio construction and investment strategy.

4.3.3 Investment and Profitability Factors

The Investment (CMA) and Profitability (RMW) factors provide crucial insights into the fundamental characteristics that drive the Magic Formula's stock selection process. A comparative analysis of these factors across the top 3500 and top 500 market capitalization segments yields a multifaceted view of the Magic Formula's performance.

For the top 3500 segment, the RMW factor presents a significant coefficient (0.167 at the 5% level), underscoring the Magic Formula's emphasis on profitable firms. This factor's significance reaffirms the strategy's core tenet: companies with high returns on capital are likely to deliver superior long-term performance. The CMA factor, while not significant, has a positive coefficient (0.054), suggesting that within this broad market segment, the Magic Formula does not distinctly favor firms based on their investment styles as aggressive or conservative.

Turning to the top 500 segment, the RMW factor exhibits an even stronger positive coefficient (0.249 at the 1% level), indicating that profitability is a predominant element in the performance of the Magic Formula within larger companies. This finding aligns with the strategy's criteria of selecting fundamentally strong companies with high earnings yields.

In contrast, the CMA factor for the top 500 segment reveals a significant negative coefficient (-0.154 at the 1% level), indicating a preference for companies engaging in more aggressive investment behaviors, characterized by higher asset growth. This may suggest that within the large-cap space, the Magic Formula successfully identifies firms that are reinvesting in their operations and growth, potentially capturing a dynamic aspect of these companies that is not fully reflected in traditional value metrics.

The divergence between the top 3500 and top 500 segments in the significance and directionality of the CMA coefficients highlights the adaptive nature of the Magic Formula strategy. While the broader market segment shows a neutral stance towards investment style, the concentrated large-cap segment indicates a clear inclination toward growth-oriented investment practices.

In summary, the contrasting dynamics of the CMA and RMW factors across market segments shed light on the strategic underpinnings of the

Magic Formula. The strategy's ability to identify and invest in profitable firms is consistent across both segments, as evidenced by the RMW coefficients. However, the approach towards investment style, as indicated by the CMA factor, varies, with a distinct bias towards aggressive investment strategies within the top 500 segment. These insights underscore the Magic Formula's nuanced application and its effectiveness in navigating different market cap environments to extract value based on profitability and growth potential.

4.3.4 Analysis of Strategy Alpha

The alpha, or intercept, in the regression model represents the excess return of the Magic Formula portfolio beyond what is predicted by the Fama and French Five-Factor model. A statistically significant alpha indicates the portfolio's ability to achieve higher returns than those accounted for by established risk factors.

For the top 3500 companies, the regression analysis yields an alpha of 1.5%, significant at the 1% level (see Table 4.1). This robust alpha suggests that the Magic Formula has effectively identified stocks that outperform the market after adjusting for market, size, value, profitability, and investment factors. The magnitude of this alpha signifies the substantial value the Magic Formula brings in identifying investment opportunities that surpass the model's expectations.

In the top 500 segment, the alpha is 1.1%, also significant at the 1% level (see Table 4.1). The existence of a significant alpha in this more narrowly focused segment suggests that the Magic Formula can consistently uncover value even among the largest, and often most analyzed, companies — where market efficiency is typically presumed to be greater. This continuity of significant alpha within a segment traditionally aligned with market efficiency propositions vividly challenges the efficient market hypothesis.

The presence of significant positive alphas across these market capitalization segments highlights the effectiveness of the Magic Formula's stock selection criteria. It emphasizes the strategy's prowess in capitalizing on inefficiencies and identifying companies with superior return potential, challenging the efficient market hypothesis's assertion that generating consistent excess returns is nearly impossible without additional risk exposure.

In summary, the observed alphas across different segments validate the Magic Formula's ability to deliver above-market returns. The evidence of significant alphas at the 1% level reinforces the strategy's credibility as a potent investment methodology that transcends the predictions of factor-based asset pricing models, offering a strategic edge to investors in various market environments.

4.3.5 Temporal Dynamics of Factor Coefficients in Value Investing

The longitudinal analysis of the Fama and French Five Factors provides valuable insights into the historical influence of these factors on the Magic Formula's performance within the top 3500 market cap segment. By examining the temporal behavior of these coefficients, we can better understand the consistency and variability of different market risks and premiums over time.

Figure 4.5 illustrates the year-on-year movement of the factor coefficients from 1998 to 2023 with error bars reflecting statistical significance. The EMR (Excess Market Return) factor consistently shows a positive coefficient, which underscores its foundational role in capturing the market's risk premium. On the other hand, the Size (SMB) factor demonstrates relative stability, indicative of a persistent, yet moderate influence on the returns, aligning with the established size premium in financial theory.

For the Value (HML) factor, there is an observable shift in the trend. It maintained its significance for a considerable duration, pointing to the traditional value premium, particularly during periods of economic downturns and recoveries. However, this factor's influence has diminished in the recent post-pandemic years, suggesting a potential realignment in the market's valuation practices, which could be reflecting a change in investor sentiment or structural market changes that affect how value is perceived and priced.

The nuanced change in the HML factor's significance in recent years invites further investigation into the long-term viability of value-based strategies in the current economic climate. It implies that market anomalies, such as those the Magic Formula seeks to exploit, may evolve, or even dissipate over time due to macroeconomic shifts or unforeseen global events like the pandemic.

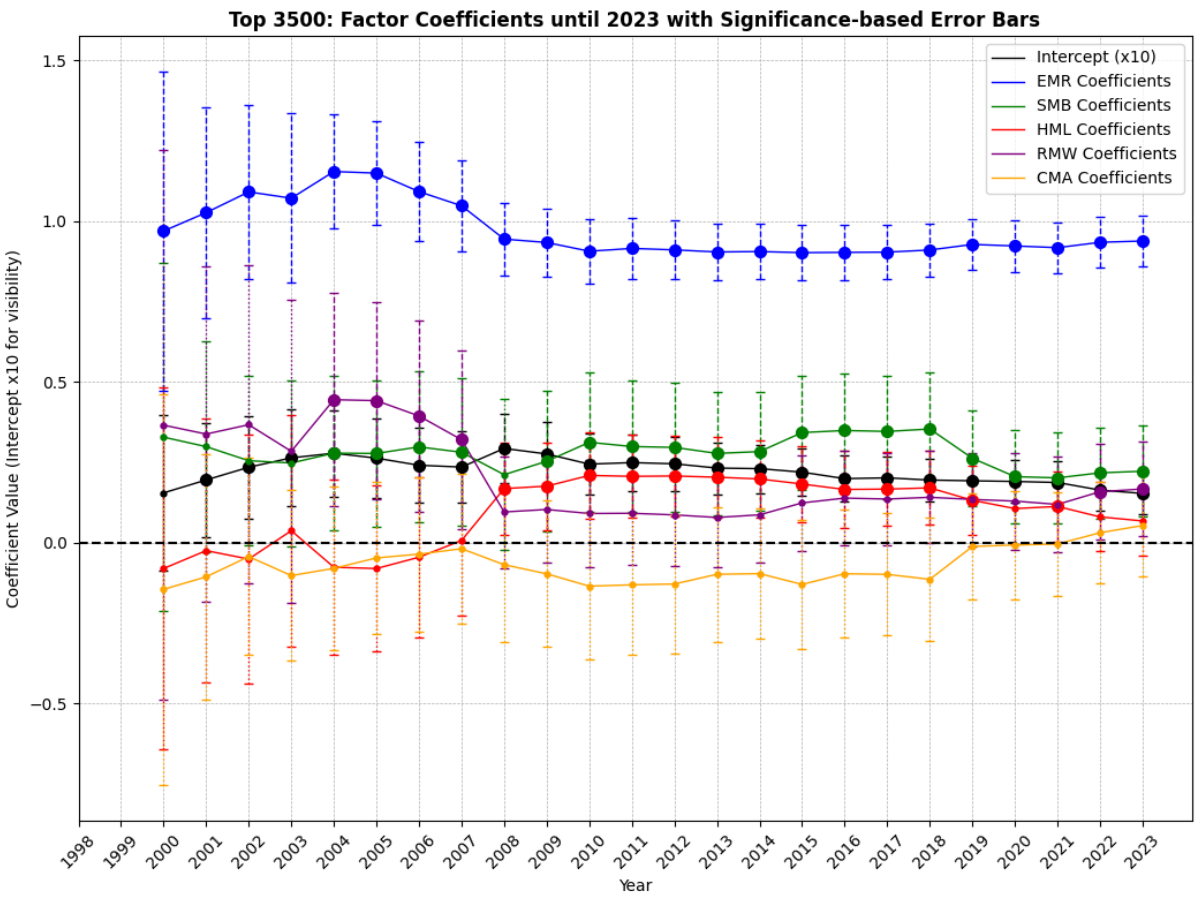

Figure 4.5: Factor Coefficients for Top 3500 with Significance-based Error Bars

For the top 500 companies, Figure 4.6 portrays the yearly evolution of the Fama and French Five Factor coefficients from 1998 to 2023. The plot reveals a distinct pattern of factor influences within this segment of larger cap companies, where the magnitudes and significances of factors diverge from those observed in the broader top 3500 cohort.

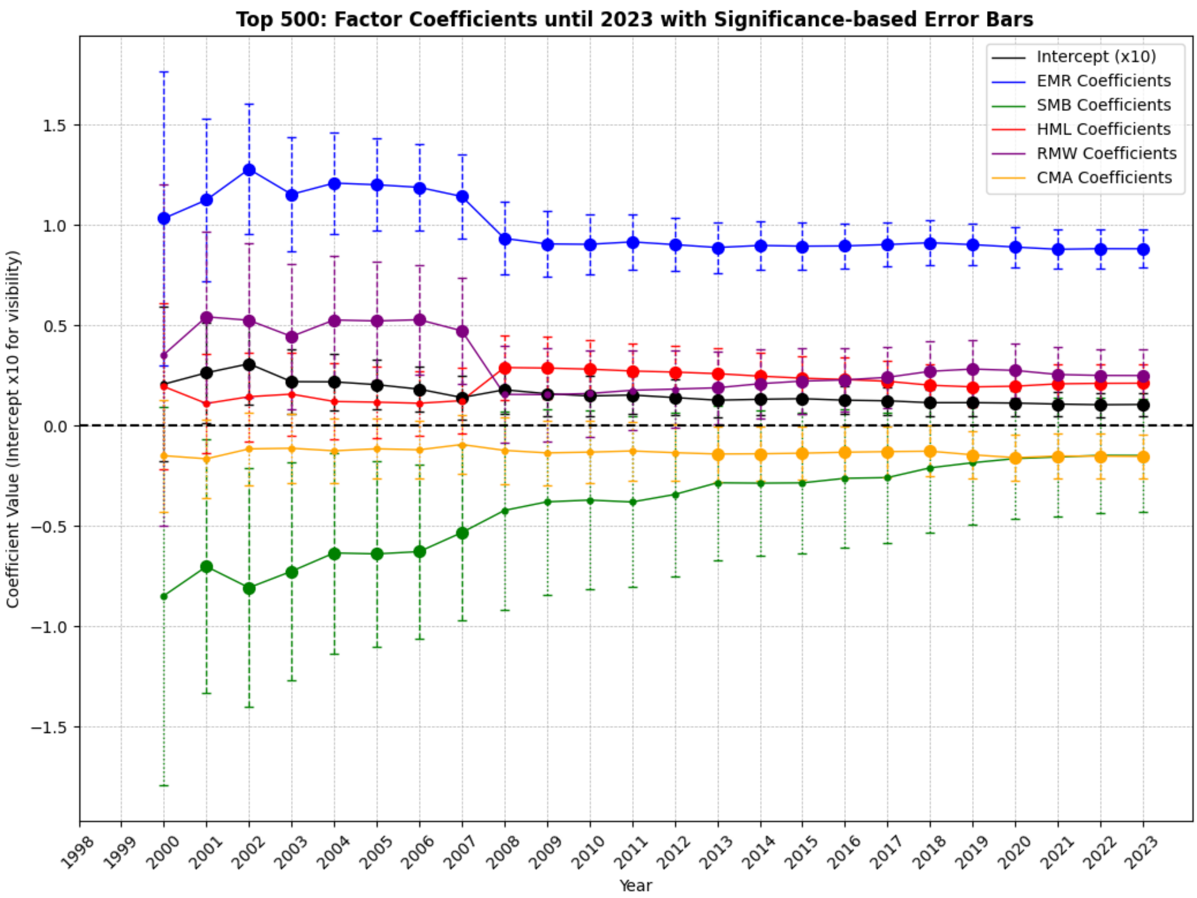

Figure 4.6: Factor Coefficients for Top 500 with Significance-based Error Bars

Turning to a more nuanced analysis, Figure 4.7 presents rolling regression coefficients for top 3500 companies until 2023, utilizing a window size that spans 16 quarters (4 years). This rolling approach allows for a dynamic assessment of the factors, providing a granular view of their fluctuating influences across economic cycles. The graphical trends reveal the immediate response of factor sensitivities to market changes, highlighting the periods when each factor's impact intensifies or wanes.

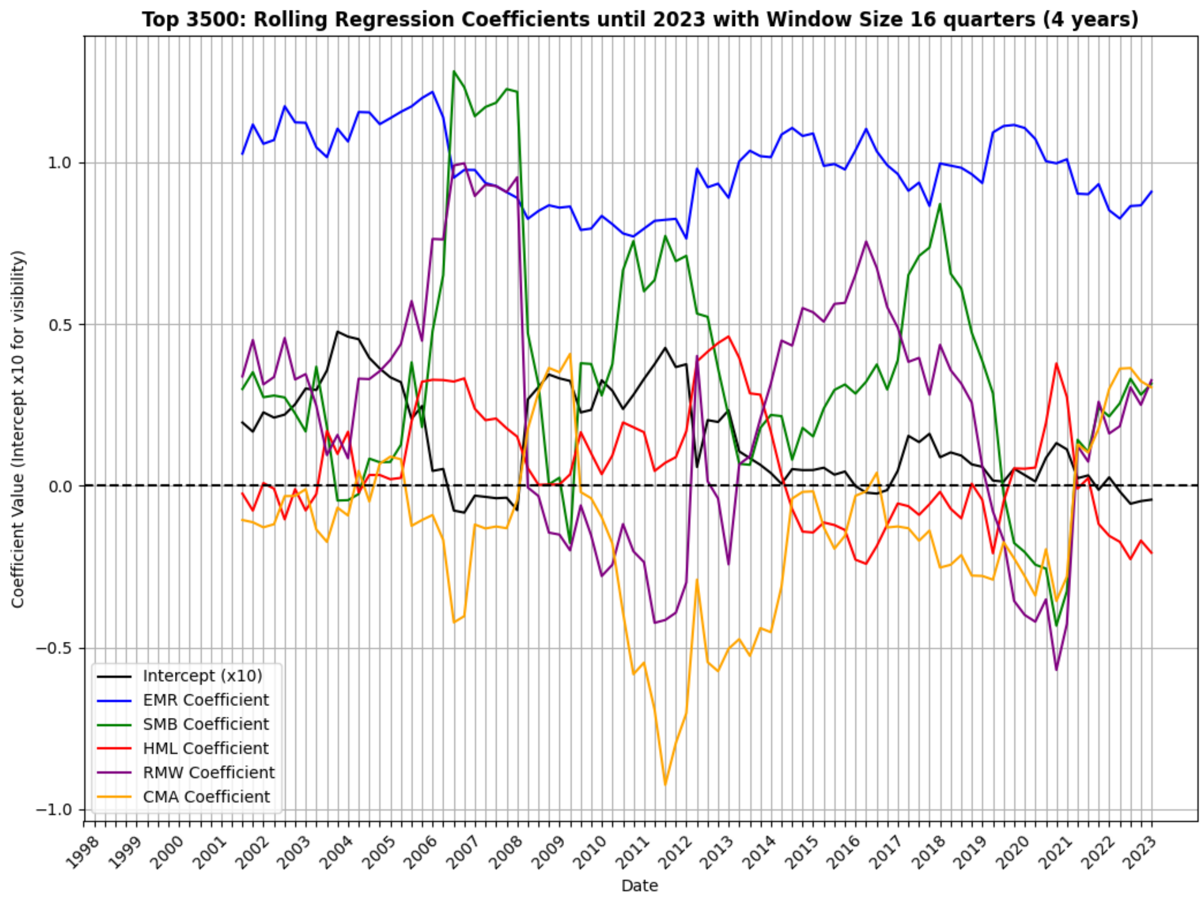

Figure 4.7: Rolling Regression Coefficients for Top 3500 Companies with Window Size 16 quarters (4 years) until 2023

Figure 4.8 delves into the dynamic behavior of the Fama and French Five-Factor model's coefficients for the top 500 companies, charted over time with a 16-quarters rolling window (4 years). This method captures the short-term fluctuations and long-term trends in factor performance, reflecting how the most substantial companies respond to economic changes and market developments.

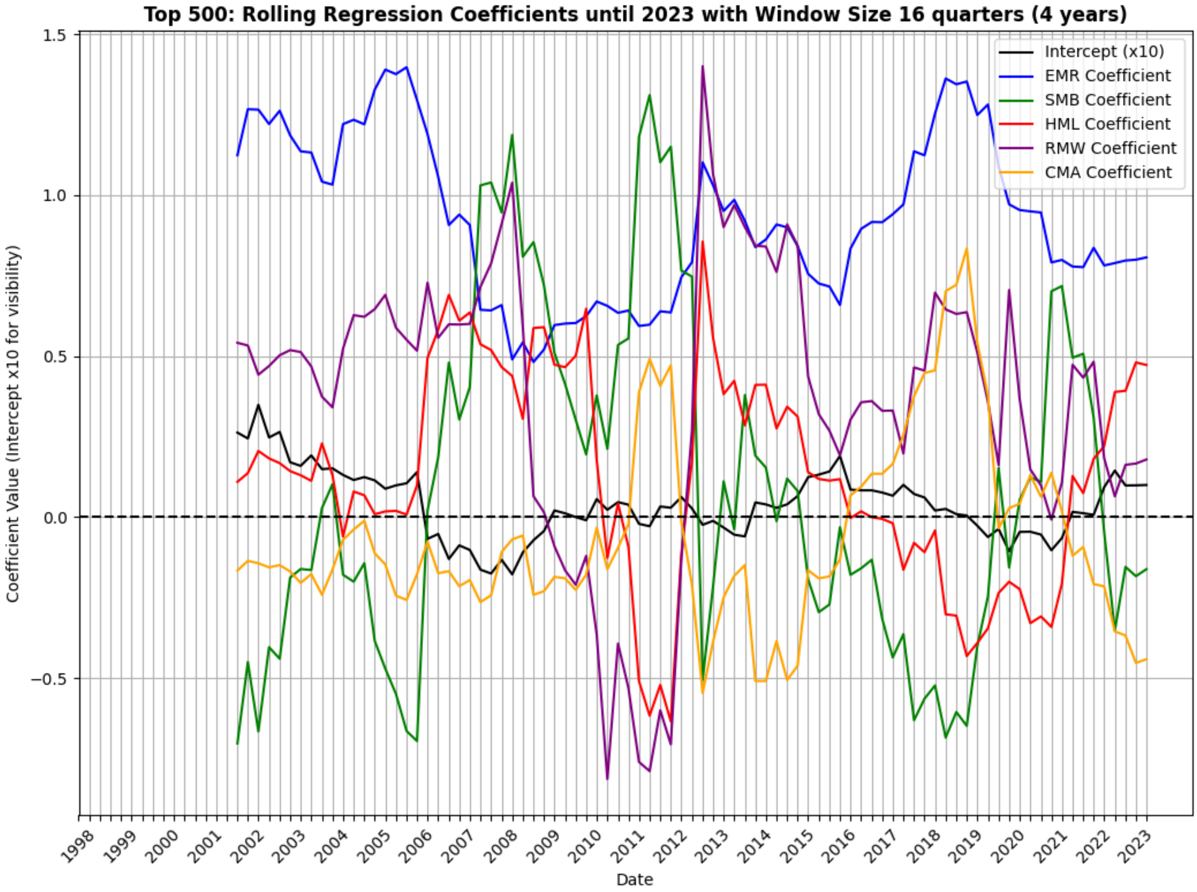

Figure 4.8: Rolling Regression Coefficients for Top 500 Companies with Window Size 16 quarters (4 years) until 2023

Together, these figures elucidate the shifting landscapes of size, value, profitability, and investment style factors in the context of value investing. The depicted trends underscore the complex nature of market forces and their interplay with investment strategies. As we consider the implications of these temporal dynamics, we gain a more comprehensive understanding of the factors that drive the success of strategies like the Magic Formula and how they adapt to an ever-changing market environment.

The analysis of these temporal patterns is crucial not only for historical understanding but also for informing future investment decisions. By recognizing the conditions under which certain factors become more prominent, investors and strategists can better tailor their approaches to align with the market's directional flows. As such, the historical study of factor coefficients becomes an indispensable component in the continuous refinement of value investing methodologies.

4.3.6 Portfolio Fit, Model Validation, and Model Efficacy

The Ordinary Least Squares (OLS) regression analysis provides a quantitative assessment of the five-factor model's fit to the Magic Formula portfolio returns in both the top 3500 and top 500 segments. An R-squared value of 0.949 for the top 3500 segment indicates that nearly 94.9% of the variability in returns is captured by the model's factors, demonstrating the model's high efficacy in accounting for systematic risk factors inherent in the portfolio's performance. Similarly, for the top 500 segment, an R-squared value of 0.863 accounts for 86.3% of the returns' variability, marking a strong, albeit slightly lower, fit compared to the top 3500 segment.

The F-statistic and its associated probability across both segments affirm the overall regression's significance, implying that the model's explanatory power is statistically robust and not due to random chance. This robustness is particularly significant for investors who rely on the model to understand the driving forces behind their returns (see Appendix A.1 and A.4 for details).

To ensure the robustness and validity of the regression analysis, a comprehensive series of diagnostic tests is conducted. These tests address potential statistical issues such as multicollinearity, heteroscedasticity, autocorrelation, and the normality of residuals within the regression models. Multicollinearity tests assess the independence of independent variables, heteroscedasticity tests examine the consistency of variance across the dataset, and autocorrelation tests look for patterns in the sequence of residuals that may affect the reliability of the regression outcomes.

Key to the regression analysis is the testing of assumptions underlying the Ordinary Least Squares (OLS) method, which includes linearity, independence, homoscedasticity, and normality of residuals. These tests are critical for confirming that the statistical inferences drawn from the regression models are valid and reliable.

Multicollinearity is assessed using the Variance Inflation Factor (VIF) to ensure that the independent variables are not highly correlated, which could distort the regression results. The VIF values for all variables were found to be below 10, indicating that multicollinearity is not a significant concern.

Heteroscedasticity is tested using the Breusch-Pagan test to verify the constancy of residual variance across the dataset. The results of the Breusch-Pagan test indicate no significant heteroscedasticity for the top 500 dataset but suggest potential heteroscedasticity for the top 3500 dataset.

Autocorrelation was evaluated using the Durbin-Watson statistic. A value of approximately 2.12 for the top 500 model suggested an absence of significant autocorrelation, aligning with the ideal range. However, the top 3500 dataset showed a statistic of 1.460, indicating potential autocorrelation issues that could affect the reliability of the model. This suggests the presence of time-series momentum or other phenomena not fully explained by the model.

Normality of the residuals was evaluated using the Shapiro-Wilk, Jarque-Bera, and Omnibus tests, as well as visualized with Q-Q plots. For the top 500 dataset, the tests indicated that the residuals were not normally distributed. However, the Q-Q plot suggested that the residuals were approximately normally distributed, with some deviations at both ends. Skewness at 0.71 and kurtosis at 3.82 confirmed these deviations, with skewness indicating a right-skewed distribution and kurtosis suggesting slightly heavier tails than a normal distribution. Overall, while there were some deviations from normality, the residuals did not exhibit extreme non-normality. For the top 3500 dataset, the Q-Q plot showed that the residuals were approximately normally distributed, aligning with the Shapiro-Wilk, Jarque-Bera, and Omnibus test results. Normality was confirmed with skewness at 0.24 and kurtosis at 3.14.

The diagnostic tests for both top 500 and top 3500 regression models were conducted to ensure the robustness and validity of the regression analysis. The results are summarized in Table 4.1.

Table 4.1: Diagnostic tests results for both the top 500 and the top 3500 regression

Diagnostic Test	**Top 500 Results**	**Top 3500 Results**
Multicollinearity (VIF)	VIF values below 10, indicating no significant multicollinearity.	VIF values below 10, indicating no significant multicollinearity.
Heteroscedasticity (Breusch-Pagan Test)	Breusch-Pagan test indicates no significant heteroscedasticity.	Breusch-Pagan test suggests potential heteroscedasticity.
Autocorrelation (Durbin-Watson Statistic)	Durbin-Watson statistic is approximately 2.12, indicating no significant autocorrelation.	Durbin-Watson statistic is 1.460, suggesting potential autocorrelation.
Normality	Shapiro-Wilk, Jarque-Bera, and Omnibus tests indicate residuals are not normally distributed. However, the Q-Q plot suggests that the residuals are approximately normally distributed, with some deviations.	Shapiro-Wilk, Jarque-Bera, and Omnibus tests indicate residuals are normally distributed.
Skewness and kurtosis	Skewness 0.71 and kurtosis 3.82	Skewness 0.24 and kurtosis 3.14.

In summary, while the regression analysis substantiates a strong fit of the Five-Factor Model to the portfolio returns examined, evidenced by substantial R-squared values and significant factor loadings, the presence of potential autocorrelation in the top 3500 segment signals a need for model refinement. Despite these areas for potential enhancement, the overall results affirm the Magic Formula strategy's efficacy in exploiting market inefficiencies and delivering risk-adjusted returns. These findings not only confirm the model’s utility in explaining performance variations but also highlight the critical role of considering multiple risk factors in the assessment of investment strategies, providing a comprehensive understanding of market behavior dynamics.

4.4 Comparative Performance Analysis

The comparative performance analysis of the Magic Formula portfolios within the top 3500 and top 500 market cap segments sheds light on the nuances of market capitalization effects, factor exposures, and the temporal stability of portfolio performance.

4.4.1 By Market Capitalization

The Magic Formula strategy's performance differentials between the top 3500 and top 500 segments are noteworthy. The top 3500 segment, with its broader market scope, has delivered a higher annualized return of 20.7% compared to the top 500's 11.3%, indicating a potential size effect where a more extensive selection universe may lead to higher performance potential. This is supported by the segment's significant exposure to the SMB factor, affirming the strategy's proclivity towards small-cap stocks that can offer higher returns.

In terms of risk-adjusted returns, the top 3500 segment exhibits a Portfolio Annualized Sharpe Ratio of 0.287, outperforming the top 500's ratio of 0.220 (see Table 4.1). This suggests that the strategy not only targets higher returns but does so with a favorable return per unit of risk, especially in the more comprehensive market segment. Additionally, the top 3500 segment's lower maximum drawdown indicates that its broader diversification might cushion against market downturns more effectively than the more concentrated top 500 segment.

4.4.2 Temporal Performance Evaluation

Temporal variations in the performance of the Magic Formula portfolios can illuminate how different market conditions have influenced their success over the study period from 1998 to 2023. Evaluating these temporal trends is essential to understand the periods when the strategy either outperformed or underperformed relative to expectations.

For the top 3500 segment, the performance dynamics reflect the influence of broader economic trends that disproportionately impact the diverse range of small to large-cap stocks. The segment has shown robust performance, regularly outperforming the market. The Excess Market Return (EMR) factor, with its consistently significant coefficient at the 1% level, highlights the critical role of market trends in influencing portfolio returns. Similarly, the Size (SMB) factor's significant impact at the 1% level underscores the importance of small-cap stock performance, indicating that market capitalization is a key performance driver during this period. Figure 4.5 reveals that the Value (HML) factor was significant from 2008 to 2019 and in 2021, marking its relevance to the segment's performance in those years. However, its significance waned in 2022 and 2023, suggesting a shift in the market's valuation approach. The Profitability (RMW) factor, significant at the 5% level, had a meaningful but comparatively moderate impact on performance, suggesting its influence is notable but not as dominant as the EMR or SMB factors. In contrast, the Conservative minus Aggressive (CMA) factor has not demonstrated a significant effect, indicating that asset growth has been less crucial in driving performance across this segment's broad market capitalization range.

The top 500 segment, predominantly consisting of large-cap stocks, illustrates a distinct pattern in factor influences. The substantial positive impact of the Profitability (RMW) factor, significant at the 1% level, underscores its crucial contribution to performance, reinforcing the notion that well-established, profitable companies are predisposed to outperform. In contrast, the significant negative coefficient for the Conservative minus Aggressive (CMA) factor at the 1% level suggests that the Magic Formula investment strategy tends to favor companies engaged in aggressive investment strategies over those with a conservative approach to asset growth. This trend reveals a market inclination towards firms that actively expand their asset base, which, despite increasing their asset-to-book value ratio, are rewarded for their growth-oriented investments.

The temporal analysis includes an evaluation of the stability of both the Sharpe and Sortino ratios for the portfolios under study across the observed period. For the top 3500 segment, a consistently higher Sharpe ratio indicates a reliable performance when adjusting for total risk, while its elevated Sortino ratio emphasizes the strategy's proficiency in minimizing the impact of negative returns, showcasing resilience during volatile market conditions. In contrast, the top 500 segment, although exhibiting a relatively lower Sharpe ratio, demonstrates a significant Sortino ratio. This highlights the segment's capability to yield higher quality returns, accentuated by a focus on profitability and effective management of downside risks. These findings underscore the differential risk-adjusted performance and downside risk mitigation between the broader and more concentrated market segments.

Overall, the temporal performance evaluation of the Magic Formula portfolios across two distinct market capitalization segments offers valuable lessons on the interplay between market conditions and investment strategy success. It highlights the necessity for investors to stay attuned to economic indicators and factor dynamics, which can significantly influence portfolio performance over time. By integrating these insights, investors can enhance their strategic approach to capture market inefficiencies and optimize risk-adjusted returns, irrespective of prevailing market cycles.

4.5 Empirical Findings on Market Anomalies

Through the empirical analysis of the Magic Formula strategy applied to the top 3500 and top 500 market cap segments, distinct market anomalies have been observed that significantly influence the strategy's performance. This section details these anomalies, emphasizing the directionality and statistical significance of factor exposures.

4.5.1 Anomalies Identified

Size Effect (SMB): The analysis for the top 3500 segment demonstrates a positive and statistically significant SMB coefficient, indicating a size effect where smaller companies within this broad segment outperform larger companies. This effect aligns with traditional financial theories positing that smaller firms tend to offer higher returns due to their growth potential and risk profile. In contrast, the SMB effect is not applicable to the top 500 segment, as this analysis focuses on larger-cap stocks, where the size effect is inherently not a factor.

Value Effect (HML): Both segments exhibit a positive HML coefficient, with statistical significance indicating that value stocks (high book-to-market ratio) outperform growth stocks (low book-to-market ratio). The positive sign of the HML factor across both segments validates the Magic Formula's efficacy in capturing the value premium, an essential component of its strategy to select undervalued companies.

Profitability (RMW): The top 500 segment shows a significant positive coefficient for the RMW factor, indicating that the Magic Formula effectively targets profitable companies, which aligns with its investment philosophy. This positive RMW factor underscores the profitability anomaly, suggesting that firms with higher operational efficiency and profitability metrics provide superior returns.

Investment Style (CMA): The negative CMA coefficient in the top 500 segment reveals an intriguing aspect of the Magic Formula's strategy, which, despite not being explicitly designed to do so, ends up favoring companies engaged in aggressive investment practices over those with conservative investment approaches. The significance of this negative coefficient at the 1% level clearly differentiates the performance and selection criteria of the Magic Formula from general market trends, highlighting its unique propensity to benefit from growth-oriented investments during the study period. This distinct alignment suggests that the strategy is particularly effective in capitalizing on specific market conditions where aggressive investments are rewarded.

4.5.2 Analysis of Anomalies

Size Effect (SMB): For the top 3500 segment, the analysis shows a positive and statistically significant SMB coefficient at the 1% level, highlighting a size effect where smaller companies outperform larger ones. This effect supports the financial theory that smaller firms tend to offer higher returns due to their growth potential and higher risk. The SMB effect is not directly applicable to the top 500 segment, as it consists of larger-cap stocks where the size differential is less pronounced, and thus, the size effect is not a significant factor.

Value Effect (HML): For the top 3500 segment, the HML factor has been significant since 2008, signifying that high book-to-market stocks outperform low book-to-market stocks, thereby reinforcing the value premium. However, its significance has waned in the last two years, suggesting a shift in the market's valuation methods or changes in investor sentiment towards value stocks. The long-term significance of the HML factor for this segment highlights its importance in selecting undervalued companies, a critical strategy of the Magic Formula. For the top 500 segment, the HML factor has consistently been significant since 2008 at the 1% level. This consistency underscores the enduring relevance of the value premium in influencing the performance of large-cap stocks, aligning with the Magic Formula's strategy to identify and invest in undervalued companies.

Profitability Effect (RMW): The RMW factor remains significant for both the top 3500 and top 500 segments. The significance at the 5% level for the top 3500 and at the 1% level for the top 500 indicates that the Magic Formula's focus on profitable companies plays a vital role in its investment philosophy. This suggests that companies with higher operational efficiencies and profitability metrics tend to offer superior returns, with a more pronounced effect observed in the top 500 segment, highlighting the importance of profitability in selecting large-cap investments.

Investment Effect (CMA): The negative CMA coefficient in the top 500 segment reveals a significant anomaly in how the Magic Formula interacts with prevailing market dynamics. Unlike the general market trend, the Magic Formula effectively identifies and benefits from companies that engage in aggressive investment patterns, typically associated with growth strategies. This differentiation is not due to an explicit preference within the formula’s strategy but is a byproduct of how these aggressive behaviors align with the formula’s selection criteria under certain market conditions. The significant negative CMA coefficient at the 1% level distinctly sets the Magic Formula apart from broader market behaviors, indicating its unique ability to leverage the specific economic climate that favors such investment approaches during the analysis period.

The analysis in this section has delineated the Magic Formula's alignment with key market anomalies, using the Fama and French Five-Factor model to assess its performance within the top 3500 and top 500 market cap segments. The evidence underscores the relevance of factor-based analyses in deciphering the complexities of investment strategies and their interaction with market dynamics. The significance and direction of factor exposures not only underscore the strategy’s ability to navigate market inefficiencies but also highlight its reliance on principles of value and profitability. These empirical findings provide a foundation for understanding the strategic nuances of the Magic Formula and its effectiveness in exploiting specific anomalies for market outperformance.

4.6 Summary of Empirical Findings

The empirical analysis has systematically assessed the performance of the Magic Formula strategy across the top 3500 and top 500 market capitalization segments, applying the Fama and French Five-Factor model over the period from 1998 to 2023. The findings provide a quantified view of the strategy's efficacy.

For the top 3500 segment, the analysis shows an annualized return of 20.7%, significantly surpassing the broader market's performance. This superior performance highlights a distinct size effect, as indicated by a positive SMB coefficient, which points towards the strategy's effectiveness in leveraging smaller-cap stocks. The HML factor, significant until recent years, demonstrates the formula's emphasis on undervalued companies, capturing the value premium efficiently. However, it's important to note the reduction in significance of the HML factor in the last two years, suggesting a potential shift in market dynamics or investor preferences.

Conversely, the top 500 segment has yielded an annualized return of 11.3%, also outperforming the general market, albeit with lower returns compared to the top 3500 segment. The lack of a significant SMB effect in this segment is consistent with its focus on larger-cap stocks, which inherently exhibit less of a size effect. Notably, the HML coefficient within the top 500 segment has remained consistently significant since 2008 at the 1% level, indicating a strong and continuous preference for value stocks. This pronounced significance of the HML factor in the top 500 segment underscores a more focused value-driven investment approach, highlighting the effectiveness of the Magic Formula in selecting undervalued large-cap stocks.

The RMW factor's positive influence on both the top 3500 and top 500 segments highlights a preference for profitable companies, notably stronger in the top 500 with higher significance and coefficients. Simultaneously, while a significant negative CMA coefficient suggests these companies are characterized by higher levels of investments relative to assets.

Risk assessment indicates that the top 3500 segment excels in delivering higher risk-adjusted returns than the top 500, as evidenced by its superior Sharpe Ratio. The top 500 segment shows slightly better resilience in market downturns, evidenced by its marginally lower maximum drawdown. However, the top 3500's efficiency in managing volatility is particularly highlighted when considering the Sortino Ratio, which underscores its effectiveness in navigating downside risk to secure significant returns.

The empirical analysis substantiates the Magic Formula's ability to navigate market inefficiencies and emphasizes the continued relevance of value and profitability in driving its strategy. The direction and significance of factor loadings, with some reaching the 1% level of significance, corroborate the robustness of the formula's underlying principles in various market conditions. These findings lay a foundation for a deeper understanding of the strategic elements that contribute to the Magic Formula's consistent outperformance in the face of market anomalies.

In analyzing the performance of the Magic Formula since 2010, a focused temporal comparison reveals that the top 3500 segment registered an annualized return of 10.8%, outperforming the market's 7.1% return. The excess return now stands at 3.7%, a decrease from the 10% observed over the last 25 years. Notably, this period has seen a pronounced shift toward large-cap technology companies, with significant investments in artificial intelligence (AI) and other innovative technologies. This industry trend is mirrored in the Magic Formula’s performance within the top 500 segment, which achieved an 11.7% annualized return, surpassing both the market's 8.8% return and the strategy’s performance within the broader top 3500 segment during the same timeframe. The consistent outperformance of the Magic Formula within the top 500 segment from 2010 to 2023 underscores the strategy’s ability to consistently select companies that yield superior risk-adjusted returns.

5 Discussion

The discussion that unfolds in this chapter is rooted in the empirical evidence amassed through a rigorous examination of the Magic Formula's application to the top 3500 and top 500 market capitalization segments from 1998 to 2023. This analysis offers a critical lens through which to view the performance of the Magic Formula in the context of the Fama and French Five Factor Model, providing answers to the research questions posited at the outset of this study. The following sections will dissect the implications of the formula's performance relative to historical benchmarks, examine the explanatory power of the Five Factor Model over this period, and extract deeper insights into the behavior of the market and the strategies it shapes. This reflective assessment aims to connect the dots between empirical findings and theoretical paradigms, offering a comprehensive understanding of the nuances uncovered in the research.

5.1 Interpretation of Empirical Findings

The meticulous examination of Joel Greenblatt's Magic Formula applied across the top 3500 and top 500 market capitalization segments has illuminated significant insights into the strategy's performance dynamics and its interaction with market factors. The analysis, spanning from 1998 to 2023, has underscored the strategy's efficacy, particularly highlighted by the robust annualized returns in the top 3500 segment. This segment's superior performance underscores the advantage of a wider market canvas, facilitating the discovery of a more extensive array of stocks that are undervalued by conventional metrics.

In contrast, the top 500 segment, despite its relatively lower performance metrics, managed to secure returns that exceed the market average. This variance in performance between the segments suggests a nuanced impact of market capitalization on the effectiveness of the Magic Formula. Specifically, it indicates that the size (SMB) and value (HML) factors, integral to the formula's selection criteria, exert differing levels of influence across broad versus concentrated market segments.

The positive SMB coefficient in the top 3500 segment underscores the strategy's effectiveness in exploiting the size premium, typically associated with higher returns among smaller-cap stocks. While the HML factor has historically indicated a preference for undervalued stocks, capturing the value premium, its significance has waned in the last two years. This suggests a potential shift in market dynamics or investor preferences, affecting the formula's reliance on the value premium within this broader market segment.

The differential performance and factor influence between the top 3500 and top 500 segments underscore the Magic Formula's adaptability and ability to extract value across different market scopes. The positive SMB coefficient in the top 3500 segment confirms the strategy's proficiency in leveraging the size premium, especially among smaller-cap stocks. Although the HML factor's significance has diminished in recent years within this broader segment, this evolution does not detract from the overall effectiveness of the Magic Formula. Instead, it highlights the strategy's resilience and its foundational principles of value investing. This adaptability suggests that, despite evolving market conditions and the complexities of modern financial markets, strategies rooted in identifying undervalued, fundamentally sound companies remain potent avenues for achieving superior returns.

The insights derived from this empirical analysis extend beyond mere numerical validation of the Magic Formula's performance. They contribute to a deeper understanding of how specific market conditions, characterized by the market capitalization of selected segments, modulate the influence of foundational financial factors on investment strategies. This nuanced interpretation of the findings sheds light on the strategic considerations necessary for applying value investing principles effectively in diverse market environments. The comprehensive evaluation of the Magic Formula demonstrates how varying market conditions, particularly different market capitalizations, influence the effectiveness of this value investing strategy, providing deeper insights into the dynamic interplay between foundational financial factors and market segments. Additionally, this analysis reveals that the Magic Formula's ability to outperform the market is not attributed to taking on more risk but rather to its strategic stock selection and effective risk management, offering critical perspectives on market efficiency and strategic investment.

5.2 Contextualization Within Financial Literature

The empirical analysis of Joel Greenblatt's Magic Formula applied to the top 3500 and top 500 market capitalization segments reveals nuanced insights into the relevance of financial market factors. Specifically, the size factor emerges as significantly influential at a 1% level in the broader top 3500 segment but not in the more concentrated top 500 segment. This distinction underscores the differential impact of company size on investment outcomes across various market scopes, illustrating how the significance of the size factor (SMB) varies between broader market segments with a lower average market capitalization and more concentrated market segments with higher market capitalization. This finding aligns with the work of Fama and French (1993), who highlighted the importance of the size premium in explaining stock returns.

Additionally, the robustness and profitability (RMW) factor shows an increase in both significance and coefficient size when moving from the top 3500 to the top 500 market capitalization segments, underscoring its critical role in the investment strategy landscape. This shift highlights the growing importance of profitability metrics in assessing investment quality among the largest companies. The increased significance of profitability aligns with recent findings in asset pricing literature, particularly the work of Novy-Marx (2013), which emphasizes the strong predictive power of profitability for stock returns. This connection suggests that the Magic Formula's focus on high return on capital and high earnings yield indirectly captures the profitability premium identified by Novy-Marx, helping to explain its ability to outperform the market.

Moreover, the investment factor (CMA) is not significant in the broader top 3500 segment, yet it becomes significantly negative at a 1% level in the top 500 segment. This suggests a preference for more aggressive investment styles within the largest market capitalization companies, contrasting with the broader market's indifference to this factor. Interestingly, Fama and French's (2017) international tests indicate that under certain conditions, large-cap stocks with aggressive investment strategies and high profitability can still yield positive returns, as evidenced by highly significant monthly gains. This observation reflects a more complex relationship between investment behavior and stock performance within large-cap stocks, aligning with the nuanced findings of the Magic Formula's performance across different market segments.

These findings highlight the complex interplay of market factors across different segments, challenging the uniform applicability of investment theories such as the Efficient Market Hypothesis (EMH) and the Fama and French Five Factor Model. The differential significance of the size, profitability, and investment factors between the top 3500 and top 500 segments suggests that market efficiency and factor effectiveness are context-dependent, emphasizing the need for investors to tailor their strategies to specific market conditions.

5.3 Theoretical Implications

This study presents a nuanced critique of the Efficient Market Hypothesis (EMH), originally proposed by Fama (1970), by demonstrating the Magic Formula's ability to systematically identify mispricing within both the top 3500 and top 500 segments. The consistent outperformance of the Magic Formula, supported by rigorous fundamental analysis, suggests that markets may not fully integrate all available information into stock prices, thus providing opportunities for investors to achieve above-average returns. This finding challenges the core assumptions of EMH and highlights the limitations of prevailing narratives around market efficiency.

Additionally, the distinct impacts of the Robust Minus Weak (RMW) and Conservative Minus Aggressive (CMA) factors, particularly within the top 500 stock segment, call for a reevaluation of traditional risk models such as the Capital Asset Pricing Model (CAPM). The CAPM's primary focus on market risk (beta) appears inadequate to fully explain the complexities of stock performance, which are significantly influenced by firm profitability (RMW) and investment behavior (CMA). The positive correlation of RMW with stock returns supports newer financial theories that advocate for a broader spectrum of risk considerations beyond traditional market risk. Conversely, the negative implications of the CMA factor, especially within larger market segments, corroborate the Fama and French Five-Factor Model (2015), which posits that conservative investment practices typically yield higher returns, while aggressive investment strategies often result in lower performance.

These observations suggest a complex interplay between a firm's financial decisions and its market valuation, underscoring the need for a more nuanced risk assessment framework that extends beyond conventional models. Such a framework would incorporate a wider array of financial activities and realities, better reflecting the intricate dynamics of market valuation and investment behavior.

Building on these insights, this study advocates for an expanded conceptualization of risk that integrates aspects of profitability and investment behavior alongside market risk. This approach aligns with the progressive developments seen in Fama and French’s models, from their seminal 1993 Three-Factor Model to their comprehensive 2015 Five-Factor Model. By challenging established financial theories and advocating for the inclusion of diverse risk factors, this study contributes to a more accurate depiction of the complex dynamics that influence market behavior.

The theoretical and practical implications of this study are substantial, prompting a reevaluation of foundational principles related to market efficiency and risk assessment. By integrating these insights into financial theory and practice, the study not only questions established norms but also aids in refining financial models to better capture the multifaceted nature of equity returns. This enriched understanding of market mechanisms enhances the robustness of financial analyses and supports more informed investment strategies, advancing our comprehension of market efficiency.

5.4 Practical Implications for Investment Strategies

This study provides valuable insights for investment practitioners, emphasizing the Magic Formula's efficacy in enhancing portfolio strategies. The Magic Formula’s demonstrated ability to consistently identify outperforming stocks across different market sizes underscores its utility as a key tool for optimizing returns in varied market environments.

The analysis of factor sensitivities, including size (SMB), profitability (RMW), and investment style (CMA), highlights the necessity for investment strategies to evolve in response to market changes. The Magic Formula's performance, particularly its ability to leverage these factors, indicates that a dynamic and informed approach to investment is essential. Notably, the value factor (HML) plays a significant role in the top 500 segment, affirming the resilience of value investing, whereas its reduced impact in the top 3500 segment points to shifting market dynamics that call for a reassessment of value-driven approaches.

In the top 500 bracket, profitability (RMW) and investment style (CMA) emerge as crucial determinants of stock performance. The emphasis on profitability underscores the importance of selecting financially robust firms, while the focus on investment style suggests favoring companies that invest aggressively in growth opportunities.

For practical application, investors are advised to adopt a flexible, factor-informed approach to portfolio management. This involves integrating the Magic Formula’s insights with a comprehensive understanding of current market trends and factor influences. By doing so, investors can tailor their strategies to be more responsive to immediate and long-term market conditions, thereby enhancing the potential for superior returns.

Implementing these strategies requires careful consideration of the multifaceted impact of various factors on stock performance. Investors should adjust their approaches based on these insights to optimize portfolio performance. This strategy not only increases the potential for achieving superior returns but also equips portfolios to better navigate the complexities of modern financial markets. Through the strategic application of the Magic Formula, augmented by a keen awareness of broader market factors, investors can achieve optimized portfolio performance that is both empirically grounded and strategically adept.

5.5 Limitations and Avenues for Future Research

This study, while extensive and robust in its analysis, acknowledges several limitations that bear on the interpretation and applicability of the findings. The analysis is dependent on historical data spanning from 1998 to 2023, which, while providing a substantial longitudinal perspective, may not fully capture current market dynamics or emerging trends that could influence investment strategies today. Additionally, the focus on the U.S. stock market limits the generalizability of the results. Financial markets vary across the globe in terms of economic conditions, regulatory frameworks, and investor behaviors, which might lead to different outcomes if the Magic Formula were applied in different contexts.

Another limitation arises from the potential sector-specific dynamics that the Magic Formula's criteria may not fully address, especially in rapidly evolving industries like technology or highly regulated sectors like utilities. These sectors can exhibit unique characteristics that significantly influence stock performance, which are not necessarily accounted for by a generalized formula. Furthermore, the use of the Fama and French Five-Factor Model, while providing valuable insights, confines the exploration to the predefined factors. Emerging factors or influences not modeled, such as geopolitical events or macroeconomic shifts, may also impact the effectiveness of the Magic Formula.

Recognizing these limitations suggests several areas for further exploration that could help validate and extend the findings of this research. Future investigations could include applying the Magic Formula to stock markets outside the U.S. to assess its adaptability and effectiveness across different economic and regulatory environments. Incorporating real-time data and conducting live trading simulations could also help evaluate the Magic Formula's performance under current market conditions, offering a more dynamic analysis of its practical applicability.

Detailed studies focusing on specific sectors could uncover how the Magic Formula performs against sector-specific risks and opportunities, particularly in industries characterized by high volatility or rapid innovation. Additionally, utilizing advanced quantitative methods such as machine learning algorithms could refine the predictive accuracy of the

Magic Formula, allowing for more sophisticated analysis and better risk management.

These potential research paths promise not only to enhance our understanding of the Magic Formula but also to contribute to broader financial knowledge, bridging the gap between theoretical models and practical investment strategies. Each of these future directions will be discussed comprehensively in the subsequent chapter, providing a roadmap for ongoing scholarship in the field of financial economics.

6 Conclusion

In this final chapter, we consolidate the extensive empirical investigation of Joel Greenblatt's Magic Formula within the context of the Fama and French Five-Factor model, as it has been applied to the US stock market from 1998 to 2023. We reflect on the key insights that have emerged, the theoretical and practical implications they carry, and the contribution this research makes to the field of finance. This conclusion serves to underscore the relevance of the findings, the study's inherent limitations, and the potential directions for future scholarship. It is a synthesis of the substantial evidence gathered and the thoughtful analysis conducted, offering a clear and comprehensive summation of the research journey and laying the groundwork for ongoing exploration in the ever-dynamic domain of financial markets.

6.1 Recapitulation of Key Insights

This thesis has undertaken a comprehensive examination of Joel Greenblatt's Magic Formula from 1998 to 2023, focusing on its efficacy within the top 3500 and top 500 market capitalization segments. The study has yielded pivotal insights, significantly enhancing our understanding of value investing methodologies in the modern financial arena.

A critical finding of this analysis is the Magic Formula's pronounced success in surpassing broader market returns, demonstrating its efficacy in stock selection based on earnings yield and return on capital.

Specifically, the top 3500 market cap segment showcased robust annualized returns of 20.7%, highlighting the advantage of a wider market canvas in facilitating the discovery of undervalued stocks. This success underscores the formula's ability to consistently identify high-performing stocks across various market conditions.

The integration of the Fama and French Five-Factor model has shed light on the nuanced roles of size (SMB), value (HML), profitability (RMW), and investment (CMA) factors in influencing the strategy's outcomes. The significant impact of excess market returns (EMR) was also identified as a key determinant. The investigation reveals a differentiated effect of these factors across market cap segments, illustrating a complex interplay where size, value, and profitability significantly affect the strategy's performance.

The analysis, as detailed in Table 4.1, found that the SMB factor's positive influence within the top 3500 segment indicates the formula's effective exploitation of size premiums, typically associated with higher returns among smaller-cap stocks. Additionally, the RMW factor demonstrates significant influence across both the top 3500 and top 500 segments, underscoring the universal importance of profitability regardless of company size. In the top 500 segment, the HML factor consistently shows significant influence, highlighting the importance of value premiums in larger companies. Conversely, the negative coefficient sign of the CMA factor in this segment suggests that an aggressive investment style is advantageous, contrasting with the typically favored conservative investment approaches.

These insights not only validate the Magic Formula's strength as a tool for value investing but also enrich the dialogue on stock market efficiency, the relevance of fundamental analysis, and the dynamic nature of factor-based investing strategies. The findings challenge existing perceptions of market efficiency, demonstrating that disciplined, value-oriented investing can exploit market inefficiencies effectively. This thesis contributes significantly to both academic understanding and practical applications in financial markets, encouraging a deeper exploration of investment strategies in the face of evolving market dynamics.

6.2 Implications and Recommendations

The empirical evidence presented in this thesis elucidates both the theoretical and practical dimensions of employing the Magic Formula in contemporary investment strategies. Theoretically, the study's findings challenge the efficient market hypothesis by demonstrating that markets may not always fully reflect available information, thereby allowing for systematic identification and exploitation of undervalued stocks. This insight invites a reevaluation of traditional market paradigms and encourages further exploration into the conditions under which market inefficiencies become pronounced and exploitable.

From a practical standpoint, the robust performance of the Magic Formula across different market capitalizations underscores its value as a versatile tool for investors aiming to enhance portfolio returns. The analysis suggests that incorporating a nuanced understanding of size, value, profitability, and investment factors can significantly augment the strategy's effectiveness. Investors are thus advised to adopt a multifaceted approach that considers these factors in concert with the core principles of the Magic Formula. This strategy not only aids in navigating the complexities of the market but also in achieving a balanced and diversified investment portfolio.

Moreover, the study's findings advocate for the integration of factor analysis into investment decision-making processes. By understanding the drivers behind the Magic Formula's success, investors can more effectively manage risk and identify opportunities for excess returns. It is recommended that investment practitioners continually monitor factor trends and economic indicators to align their strategies with evolving market conditions.

For academics, this research opens several avenues for future inquiry, including the exploration of the Magic Formula's applicability in international markets and its performance relative to alternative investment strategies. Such investigations could further delineate the boundaries of market efficiency and the practical utility of value investing principles in a global context.

In light of these insights, investment professionals, particularly portfolio managers and financial advisors, are encouraged to leverage the findings of this study to refine their investment strategies. Emphasizing the importance of a dynamic, informed approach to stock selection, this thesis offers a compelling case for the integration of fundamental analysis and factor considerations in pursuit of superior investment outcomes.

6.3 Future Research

Exploring the Magic Formula's application across a broader spectrum of global markets could significantly enrich future research. Assessing how different economic conditions affect the formula's effectiveness could yield insights into its adaptability and robustness under various financial systems. Additionally, sector-specific analyses might uncover critical data on how industry-specific factors influence the Magic Formula's performance. For instance, sectors like technology, characterized by rapid innovation, and utilities, known for stability and lower volatility, could provide distinct insights into sectoral investment dynamics.

Expanding the Fama and French Five-Factor Model to include additional factors could notably enhance our understanding of investment strategy performance, particularly the Magic Formula's notable market outperformance. Introducing a momentum factor, as advocated by Carhart (1997), could provide deeper insights into trends influencing stock performance, capturing the propensity of stocks to continue performing in alignment with their recent trajectory. This adjustment could potentially elevate the model's ability to explain variations in returns, aligning more closely with observed market behaviors.

Similarly, integrating a 'Stable Minus Volatile' (SMV) factor, inspired by Robert Haugen's research, could offer significant improvements in the model's predictive accuracy. Haugen's findings suggest that lower volatility stocks tend to outperform their higher volatility counterparts (Haugen, 1995; 2004). This challenges conventional financial theories that equate higher risk with higher returns. By quantifying the performance differentials between low and high volatility stocks, the SMV factor would not only reflect Haugen's empirical observations but also provide a nuanced understanding of risk and return dynamics. This enhancement would allow the model to better capture the real-world complexities of market behavior, potentially leading to more robust investment strategies.

Methodological advancements could also play a critical role in future research. Employing sophisticated econometric techniques, such as machine learning algorithms for predictive analysis or network theory to comprehend interdependencies in financial markets, could deepen insights and enhance the robustness of research findings.

Lastly, fostering ongoing dialogue within the academic and investment communities regarding these findings and future investigations will be crucial for refining and evolving investment strategies. This collaborative approach ensures that theoretical advancements align with practical investment needs, bridging the gap between academic research and real-world applications.

6.4 Reflective Conclusion

This thesis marks a significant exploration into the efficacy of Joel Greenblatt's Magic Formula, affirming its stature as a powerful tool for disciplined, value-oriented investing. Over a comprehensive analysis spanning twenty-five years, this study has not only confirmed the formula's capability to surpass broader market indices but has also deepened our understanding of the complex dynamics that drive stock market performance.

The insights derived from this research extend well beyond academic discussions, providing practical, actionable strategies for finance professionals. By integrating rigorous fundamental analysis with a keen awareness of market factors, this study highlights the potential to craft investment strategies that effectively capitalize on market inefficiencies. This approach challenges prevailing assumptions about market efficiency and encourages a reevaluation of how markets assess asset values.

Furthermore, this thesis contributes to bridging the gap between theoretical finance and practical investment decision-making. The consistent success of the Magic Formula across varied market segments demonstrates the enduring applicability of value investing principles, even in today's volatile and complex financial environment.

The investigation of the Magic Formula has revealed intricate interactions among market efficiency, factor influences, and the performance of investment strategies. It establishes a robust framework for understanding how disciplined approaches to stock selection can lead to superior investment returns. This framework not only enhances the toolkit available to investors navigating the uncertainties of the stock market but also serves as a foundational reference for ongoing financial research.

As the financial landscape continues to evolve, the insights from this study will undoubtedly prove invaluable for future academic inquiries and practical investment strategies. They highlight the persistent relevance of fundamental analysis in securing investment success, promoting a deeper comprehension of market mechanics and contributing to the broader field of finance.

In conclusion, while this study has provided substantial insights into value-oriented investing, it also opens several avenues for future research. Investigating the Magic Formula's effectiveness in international markets and its comparison with other investment strategies could further clarify the boundaries of market efficiency and the global applicability of value investing principles. This ongoing exploration will continue to enhance our understanding of financial markets and refine investment strategies tailored to diverse economic conditions.

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Appendices

The appendices of this thesis furnish a comprehensive foundation for the study's empirical analysis, systematically detailing the data and methodologies that underpin the findings. Central to this section are the Ordinary Least Squares (OLS) regression summaries for the investigated market segments, covering the top 3500, top 2500, top 1500, and top 500 companies over the span from 1998 to 2023. These summaries are presented as appendices A.1 to A.4, revealing the nuanced performance dynamics of Joel Greenblatt's Magic Formula across varied levels of market capitalization and providing a statistical basis for the research conclusions.

Further, appendices A.5 to A.12 extend the empirical findings with a series of visual representations, including comparative analyses of rolling and accumulated returns, the Sharpe Ratio, and factor coefficients with significance-based error bars. These visual appendices complement the narrative of Chapter 4, "Empirical Analysis," by offering a deeper comparative analysis and visual insights into the Magic Formula's performance not only for the primary focus segments of the top 3500 and top 500, but also across the top 2500 and top 1500 market segments. This broader examination enhances the research with a more layered understanding of how the Magic Formula fares across a wider spectrum of market capitalizations.

Appendix A.1: Top 3500 OLS Regression Results

Top 3500

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.949
Model:                            OLS   Adj. R-squared:                  0.946
Method:                 Least Squares   F-statistic:                     358.2
Date:                Wed, 21 Feb 2024   Prob (F-statistic):           1.97e-60
Time:                        11:54:36   Log-Likelihood:                 222.70
No. Observations:                 102   AIC:                            -433.4
Df Residuals:                      96   BIC:                            -417.6
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0153      0.003      4.824      0.000       0.009       0.022
EMR            0.9385      0.040     23.717      0.000       0.860       1.017
SMB            0.2223      0.071      3.117      0.002       0.081       0.364
HML            0.0677      0.054      1.252      0.214      -0.040       0.175
RMW            0.1673      0.074      2.249      0.027       0.020       0.315
CMA            0.0535      0.079      0.676      0.501      -0.104       0.211
==============================================================================
Omnibus:                        1.444   Durbin-Watson:                   1.460
Prob(Omnibus):                  0.486   Jarque-Bera (JB):                1.047
Skew:                           0.238   Prob(JB):                        0.592
Kurtosis:                       3.142   Cond. No.                         36.3
==============================================================================
```

Appendix A.2: Top 2500 OLS Regression Results

Top 2500

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.937
Model:                            OLS   Adj. R-squared:                  0.934
Method:                 Least Squares   F-statistic:                     285.8
Date:                Wed, 21 Feb 2024   Prob (F-statistic):           5.29e-56
Time:                        12:42:48   Log-Likelihood:                 229.62
No. Observations:                 102   AIC:                            -447.2
Df Residuals:                      96   BIC:                            -431.5
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0118      0.003      3.999      0.000       0.006       0.018
EMR            0.9225      0.038     24.089      0.000       0.846       0.999
SMB            0.1647      0.075      2.185      0.031       0.015       0.314
HML            0.0225      0.047      0.484      0.630      -0.070       0.115
RMW            0.2044      0.067      3.037      0.003       0.071       0.338
CMA           -0.0093      0.073     -0.127      0.899      -0.154       0.136
==============================================================================
Omnibus:                        5.410   Durbin-Watson:                   1.767
Prob(Omnibus):                  0.067   Jarque-Bera (JB):                7.983
Skew:                           0.064   Prob(JB):                       0.0185
Kurtosis:                       4.365   Cond. No.                         37.2
==============================================================================
```

Appendix A.3: Top 1500 OLS Regression Results

Top 1500

OLS Regression Results

Dep. Variable:	y	R-squared:	0.934
Model:	OLS	Adj. R-squared:	0.931
Method:	Least Squares	F-statistic:	273.6
Date:	Wed, 21 Feb 2024	Prob (F-statistic):	3.77e-55
Time:	12:44:15	Log-Likelihood:	248.11
No. Observations:	102	AIC:	-484.2
Df Residuals:	96	BIC:	-468.5
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0087	0.002	3.743	0.000	0.004	0.013
EMR	0.9129	0.035	26.377	0.000	0.844	0.982
SMB	-0.0287	0.093	-0.308	0.758	-0.213	0.156
HML	0.1281	0.039	3.281	0.001	0.051	0.206
RMW	0.2139	0.051	4.161	0.000	0.112	0.316
CMA	-0.1067	0.058	-1.855	0.067	-0.221	0.007

Omnibus:	4.793	Durbin-Watson:	1.840
Prob(Omnibus):	0.091	Jarque-Bera (JB):	4.219
Skew:	0.391	Prob(JB):	0.121
Kurtosis:	3.617	Cond. No.	46.2

Appendix A.4: Top 500 OLS Regression Results

Top 500

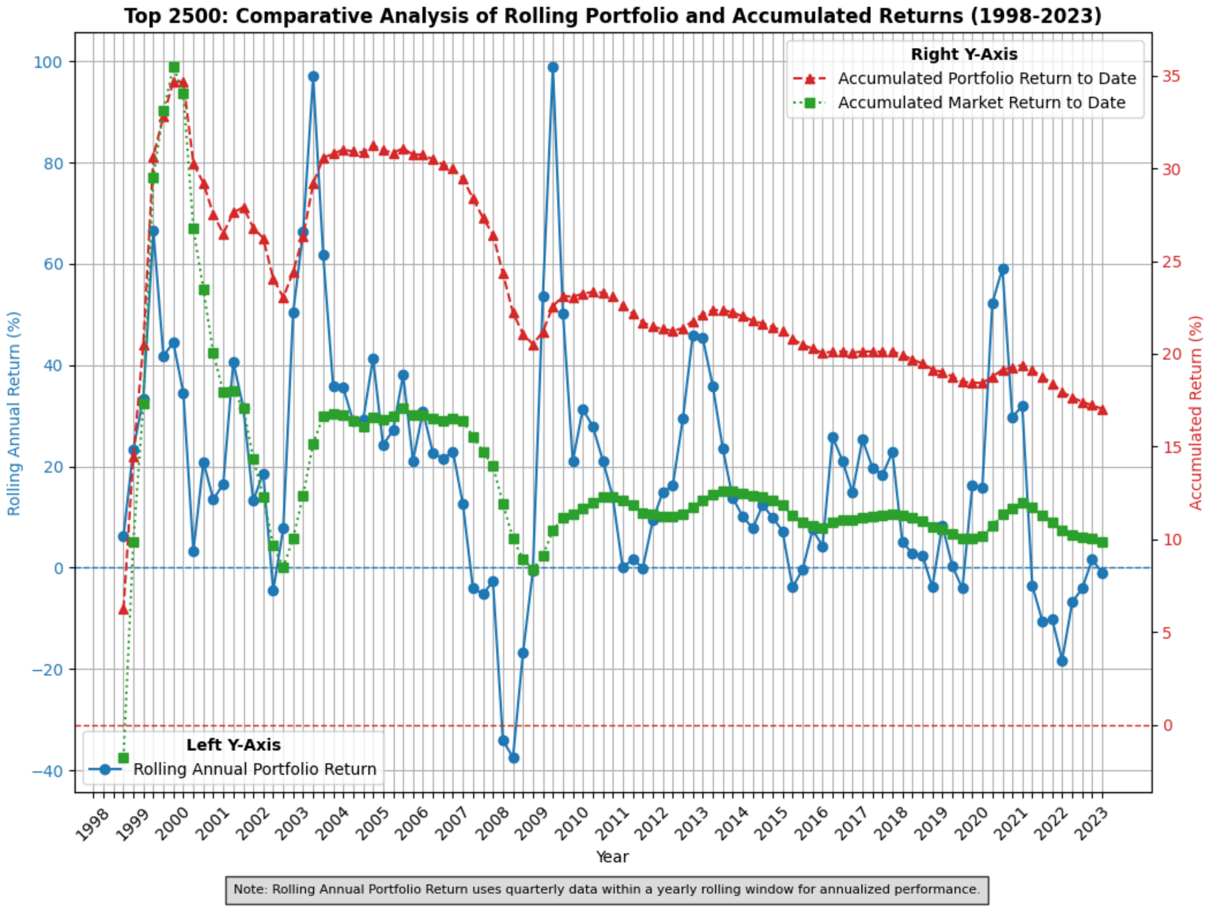
OLS Regression Results

Dep. Variable:	y	R-squared:	0.863
Model:	OLS	Adj. R-squared:	0.856
Method:	Least Squares	F-statistic:	120.7
Date:	Wed, 21 Feb 2024	Prob (F-statistic):	8.20e-40
Time:	12:45:06	Log-Likelihood:	224.78
No. Observations:	102	AIC:	-437.6
Df Residuals:	96	BIC:	-421.8
Df Model:	5		
Covariance Type:	nonrobust		

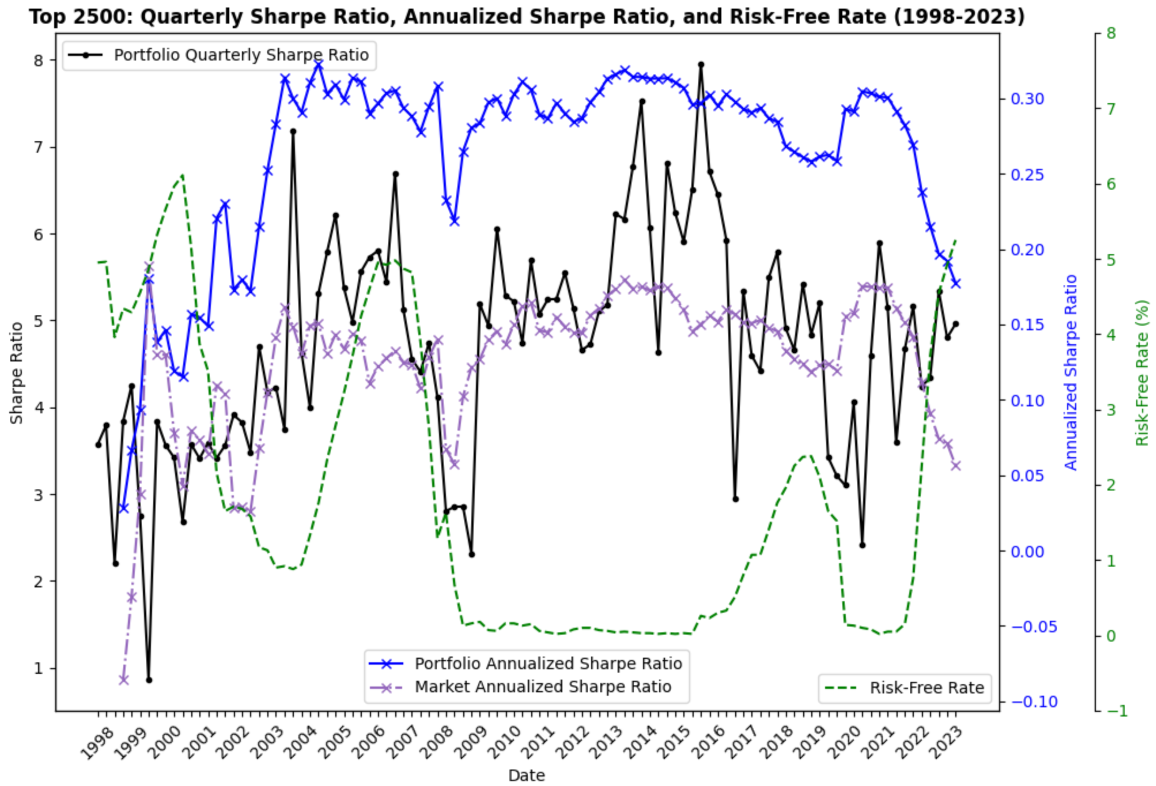
	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0105	0.003	3.540	0.001	0.005	0.016
EMR	0.8805	0.048	18.377	0.000	0.785	0.976
SMB	-0.1491	0.141	-1.057	0.293	-0.429	0.131
HML	0.2108	0.047	4.503	0.000	0.118	0.304
RMW	0.2490	0.065	3.823	0.000	0.120	0.378
CMA	-0.1537	0.055	-2.769	0.007	-0.264	-0.044

Omnibus:	11.072	Durbin-Watson:	2.119
Prob(Omnibus):	0.004	Jarque-Bera (JB):	11.452
Skew:	0.710	Prob(JB):	0.00326
Kurtosis:	3.823	Cond. No.	53.1

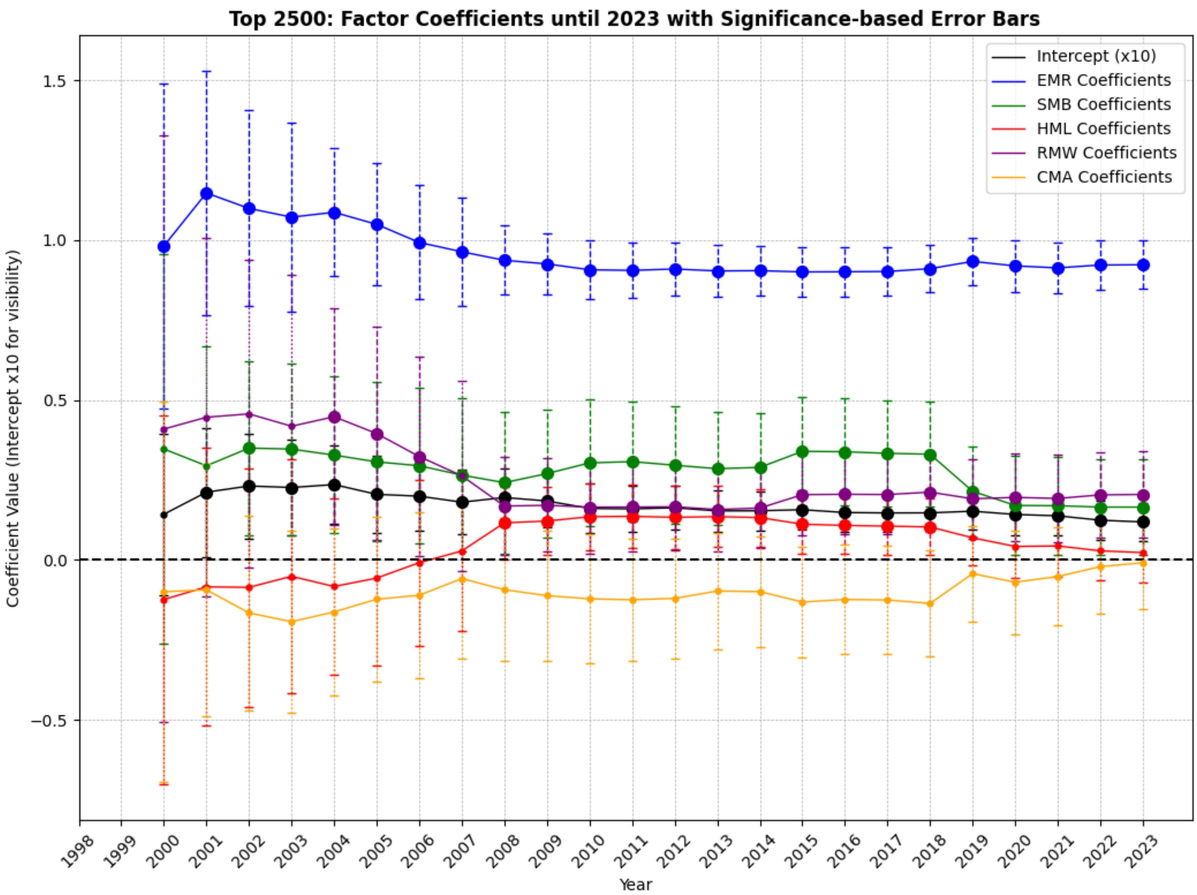
Appendix A.5: Comparative Analysis of Rolling Annual Portfolio Returns (left y-axis) shown with Accumulated Portfolio and Market Returns (right y-axis) for Top 2500 companies (1998-2023).



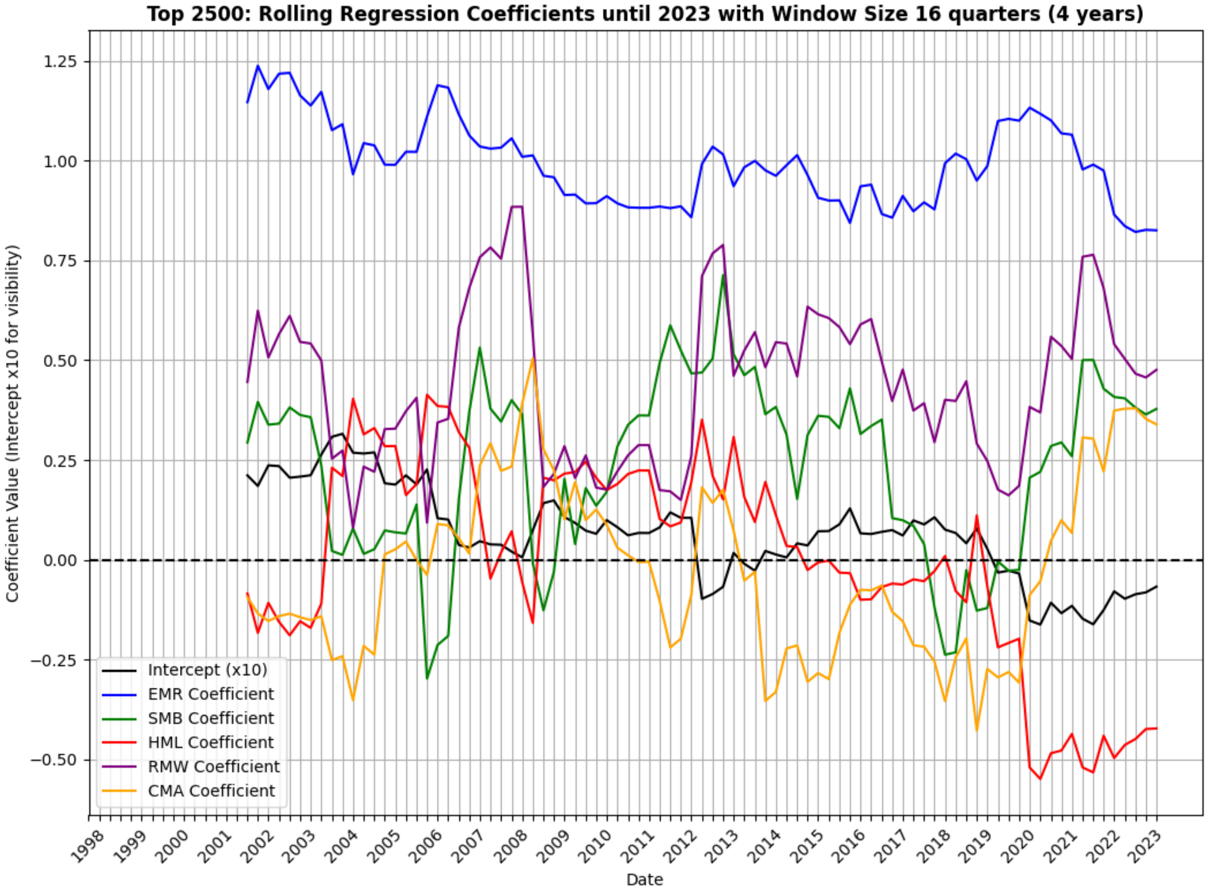
Appendix A.6: Quarterly Sharpe Ratio (left y-axis) shown with Portfolio and Market Annualized Sharpe Ratio (right y-axis) for Top 2500 companies (1998-2023)



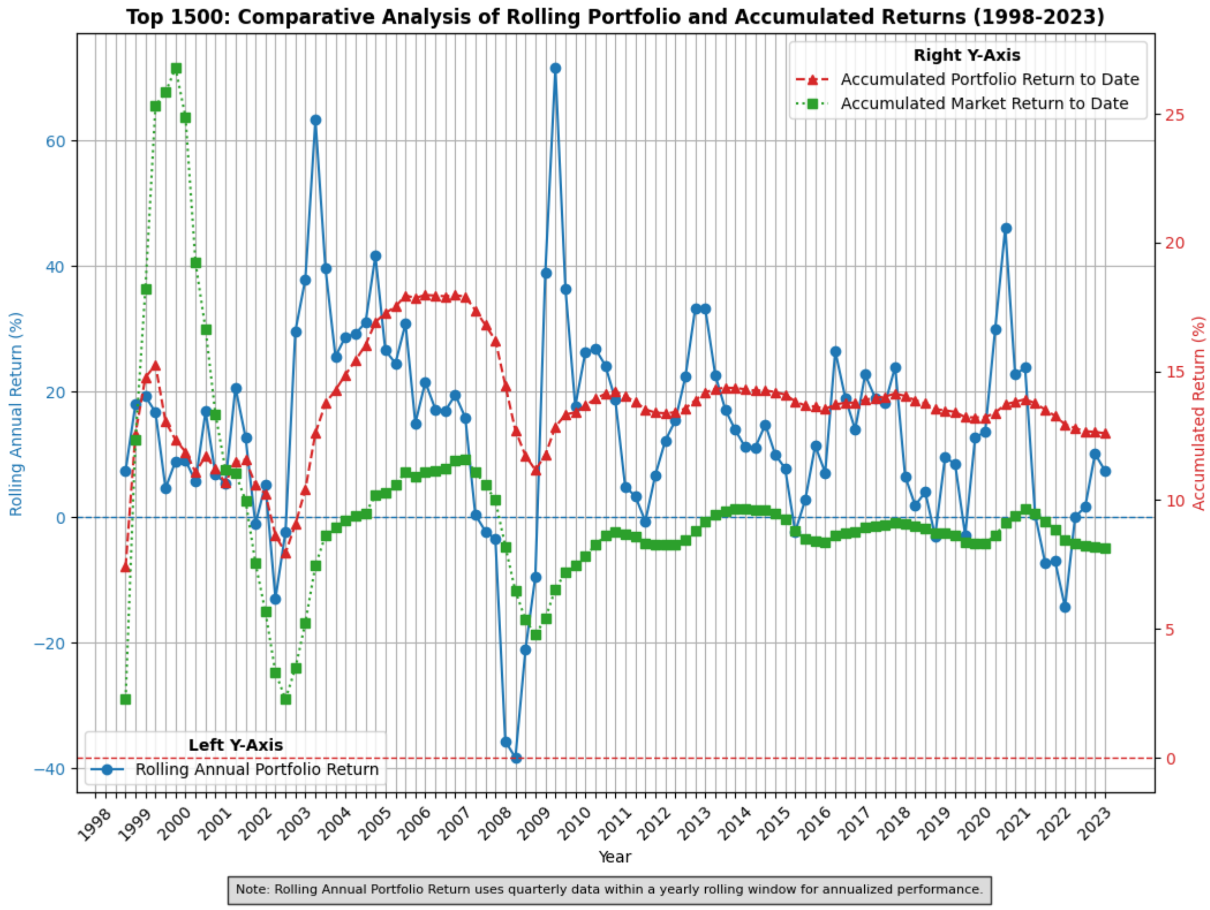
Appendix A.7: Factor Coefficients for Top 2500 with Significance-based Error Bars



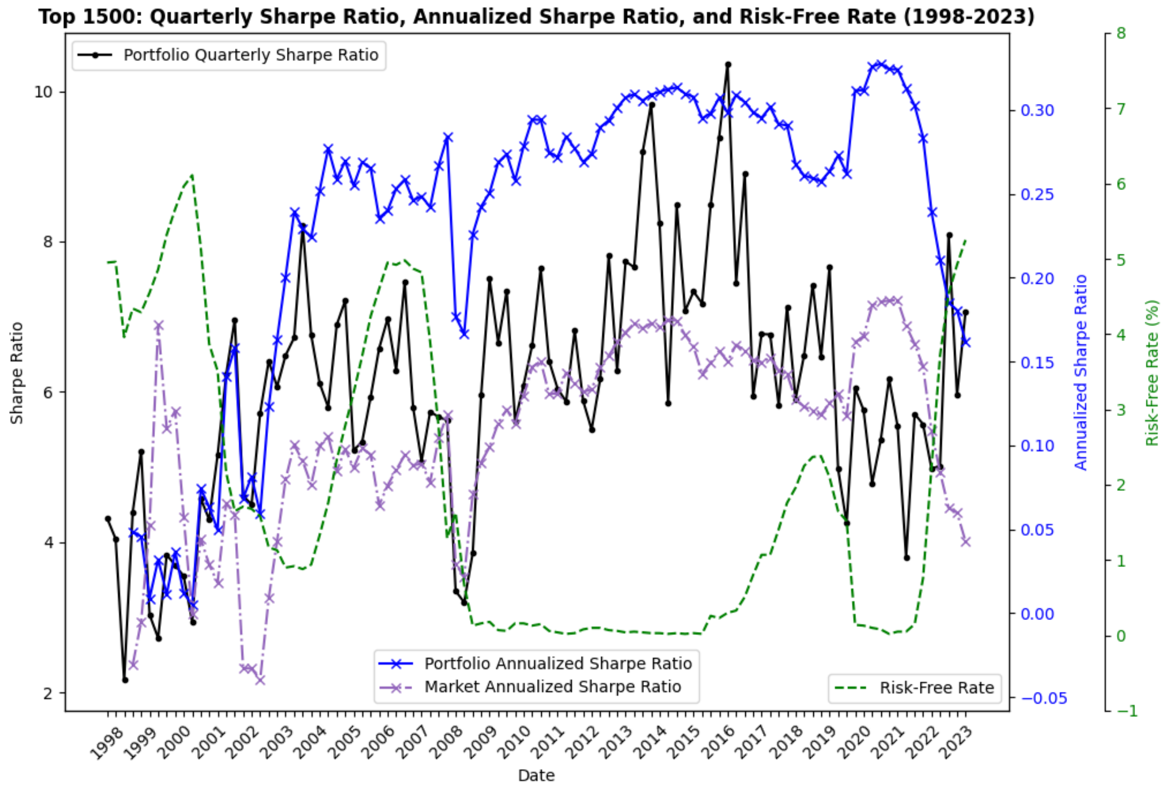
Appendix A.8: Rolling Regression Coefficients for Top 2500 Companies with Window Size 16 quarters (4 years) until 2023



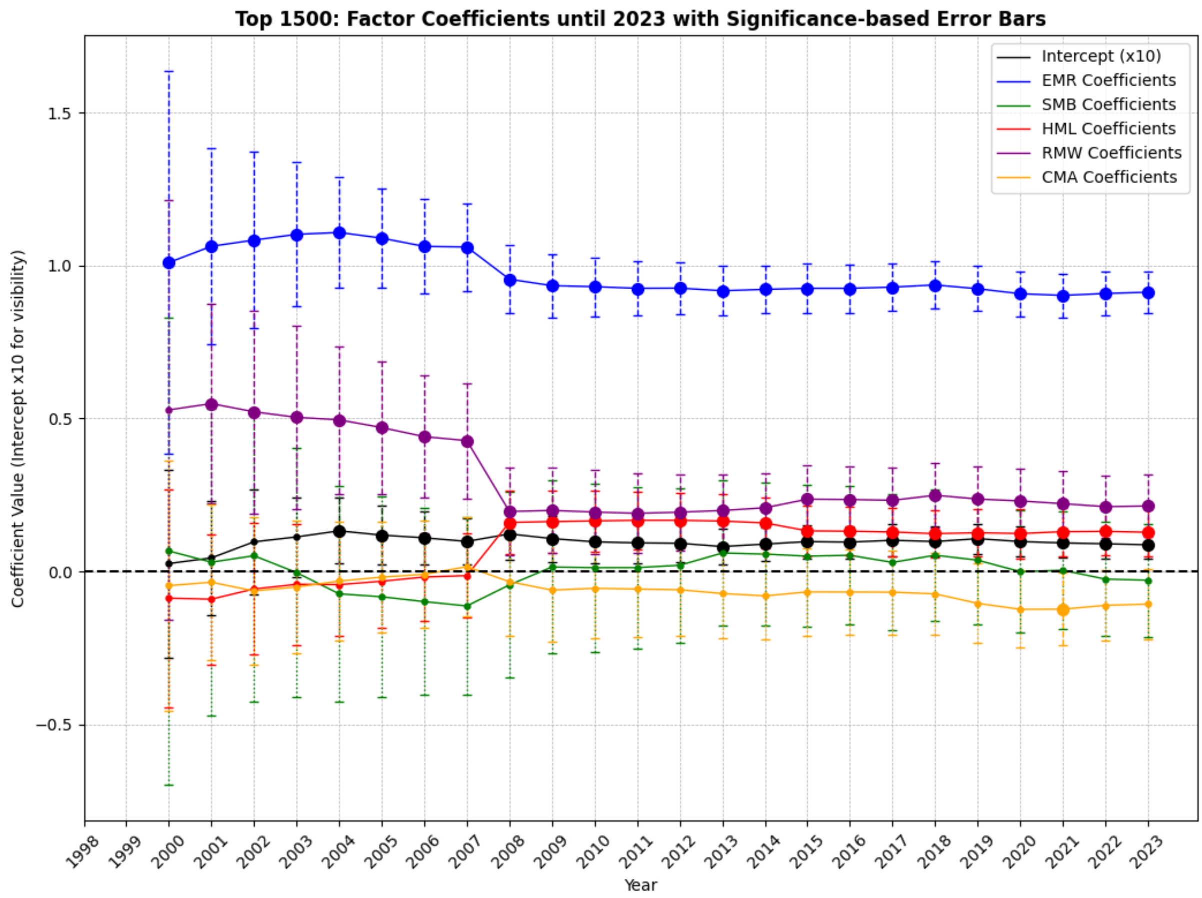
Appendix A.9: Comparative Analysis of Rolling Annual Portfolio Returns (left y-axis) shown with Accumulated Portfolio and Market Returns (right y-axis) for Top 1500 companies (1998-2023).



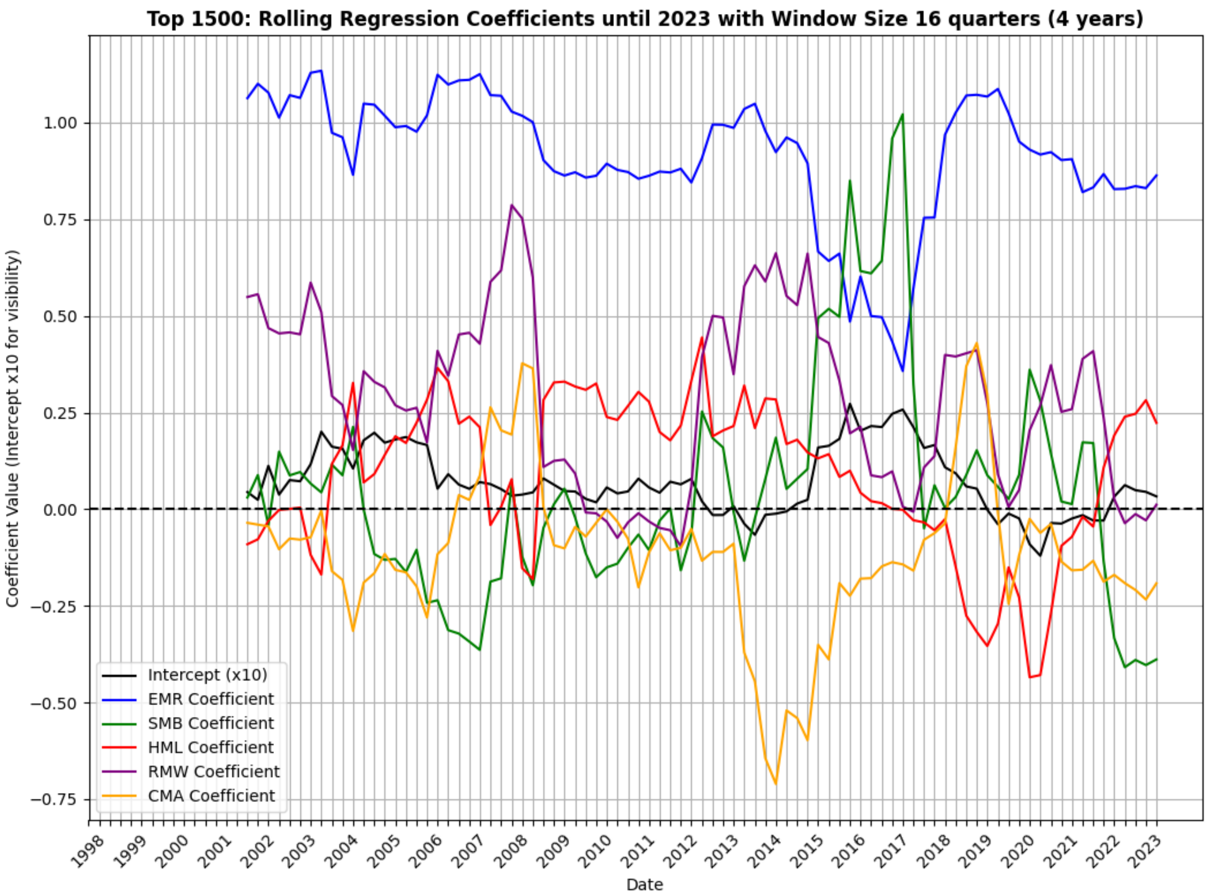
Appendix A.10: Quarterly Sharpe Ratio (left y-axis) shown with Portfolio and Market Annualized Sharpe Ratio (right y-axis) for Top 1500 companies (1998-2023)



Appendix A.11: Factor Coefficients for Top 1500 with Significance-based Error Bars



Appendix A.12: Rolling Regression Coefficients for Top 1500 Companies with Window Size 16 quarters (4 years) until 2023



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